Managerial Beliefs and Incentive Policies*

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Abstract

This article examines incentive contracts under moral hazard when a principal and agents disagree about the likelihood that a task succeeds. The direction of disagreement alters the effectiveness of monetary incentives. The principal’s optimal contract is relative performance evaluation when she is more optimistic than the agents, and joint performance evaluation when she is less optimistic. We further show why disagreement may prevail in organizations by considering a simple job assignment problem.

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1 Introduction

Organizational policies are largely shaped by managers’ life experiences, values, and beliefs. When managers have their own subjective beliefs about the prospect of a new project, their beliefs directly affect managerial decisions. In particular, in this paper, we study the effects of a manager’s beliefs on incentive structures, which we believe is one of most important decisions made by managers in organizations. For example, Jack Welch, former CEO of General Electric, has been known as an optimist, and also known for championing internal competition. He was one of the most famous practitioner of forced ranking system.\(^1\) We believe that his optimism is deeply related to his management style and incentive policies.

When a manager does not share her beliefs about contracting environment with workers, what kinds of incentive schemes and compensation arrangements does she offer to the workers? Are incentives provided collectively based on team performance or competitively based on relative performance? How does a manager want to assign workers to tasks according to her beliefs relative to the workers’?

To address these questions, we extend the standard moral hazard model in which a principal offers an incentive contract to agents. The likelihood that a task succeeds is determined not only by agents’ unobservable effort, but also by their beliefs about a working environment. A manager and agents assess the working environment differently, and thus have disagreement on the likelihood that the tasks succeed. This is the key element of the model – the contracting parties have their own subjective beliefs on the probability of success. Although they have different prior beliefs, they do not update their beliefs when they become aware of the difference. They simply agree to disagree.\(^2\)

When a principal contracts with a single agent, the principal with less optimistic outlook provides higher-powered incentives. The principal with weak beliefs about the success of a task expects that it is less likely to pay compensation even if she promises to give high-powered incentives. When a principal interacts with two agents and offers an interdependent compensation structure, the contractual outcomes are significantly different. The optimal incentive contract follows relative performance evaluation (hereafter RPE) when a manager is rather optimistic, whereas joint performance evaluation (hereafter JPE) is preferred when a manager is rather pessimistic.

The intuition behind these results is as follows. With two agents, the incentives can be provided as a mix of two devices: compensation for joint performance and compensation for relative performance. The principal’s optimal mix of the two is determined by balancing the tradeoff between the principal’s relative price of the two and the agents’ relative responsive-

ness to the two. The balance is in turn dependent on different beliefs among the contracting parties. In particular, when a principal is more optimistic than her agents, she thinks that compensation for joint performance is more likely to be made than the agents think. Conversely, the agents think that compensation for relative performance is more likely to be received than the principal thinks. As a result, a principal offers the RPE wage scheme. The opposite is true when a principal is less optimistic than agents – a principal offers the JPE wage scheme.

We then examine a principal’s job assignment problem. We find that a principal prefers to have the agents with different beliefs from herself for a task. This may explains why disagreement is so commonly observed in organizations. Disagreement is naturally emerging from organizational management such as project choice, recruiting, and job assignment.

There is a growing literature that examines the contractual outcomes when contracting parties have their own subjective beliefs about market environment. The literature has been largely concerned with the effects of the agent’s biased beliefs on contractual outcomes. Otto (2014) and many empirical papers investigate the effects of the CEO’s optimism on executive compensation. Several papers study the effects of different prior beliefs on incentive contracts under moral hazard. In particular, Santos-Pinto (2008) and de la Rosa (2011) are closely related to ours.

de la Rosa (2011) considers the case with one principal and one agent, and allows heterogeneous beliefs in two dimensions – an agent can overestimate the probability of success for any given effort level and the marginal contribution of his effort to the probability of success. The former is referred to as optimism, while the latter is referred to as overconfidence. He studies the effects of each type of bias on the intensity of incentives, and explains when an equilibrium contract entails higher or lower-powered incentives. Santos-Pinto (2008), on the other hand, considers the approach of Mookherjee (1984) with a principal-multiple-agent model, and allows agents to have mistaken beliefs about each other’s ability. He shows that an optimal contract offers an interdependent incentive scheme. Like Santos-Pinto (2008), our focus is on the multiple-agent case. But we take the simpler approach of Che and Yoo (2001) by which JPE or RPE can be characterized formally, and show that the optimal incentive structure depends on the direction of disagreement between principal and agents. In addition, we show that the principal’s expected payment is not monotonic with the agents’ beliefs, and explore implications about job assignments.

In this literature, it is assumed that the principal has an unbiased belief and offers a contract that is able to exploit the agent’s biased beliefs. This paper, however, focuses on the implications of a principal’s beliefs on incentive policies. In this respect, the closest paper is Van den Steen (2005). He considers that a manager can have an important influence on a firm’s behavior by implementing a project generated by an employee. Our paper, however, considers a manager’s choice of incentive policies when there exists moral hazard. The sorting outcome is also largely different – in his model, strong managerial beliefs attract employees with similar
beliefs, whereas managers want to assign workers with different beliefs in our model.

Our paper also makes a contribution to the literature that has studied the merits of JPE and RPE (Green and Stocky 1983; Mookherjee 1984; Itoh 1991; Che and Yoo 2001; Kvaloy and Olsen 2006, Kim 2012; Fleckinger and Roux 2012). To the best of our knowledge, our model first points out that different beliefs between contracting parties may determine the emergence of JPE and RPE as an optimal incentive scheme.

The remainder of the paper is organized as follows. Section 2 lays out the basic model. Section 3 characterizes the incentive contracts. We begin with the benchmark case in which a manager contracts with one agent. Then, we study the two-agent case and present the main results. In Section 4, we extend the model by considering the manager’s effort selection problem, continuous effort, and heterogeneous beliefs among agents. Section 5 concludes.

2 Model

Consider that a risk neutral manager (principal) employs a risk averse worker(s) (agents) to perform a task. The outcome depends on the worker’s effort $e \in \{0, 1\}$, which cannot be observed by the manager as in a standard moral hazard model. The worker’s cost function is given by $C(e) = ce$. That is, the worker’s supply of effort, $e = 1$, incurs a utility cost $c$, whereas shirking, $e = 0$, does not incur such a cost. The model with binary effort simplifies the analysis, but does not affect the qualitative nature of our results, as will be shown in Section 4.2. The worker has an outside option which is given by $u_0$.

The outcome of the task can be either success or failure, $x \in \{S, F\}$, which is observable and verifiable. For the task to be successfully accomplished, the worker should exert effort and a working environment must be good as well. The manager and the worker have their own subjective beliefs about the probability that the working environment is good: $\theta_m \in (0, 1)$ and $\theta_w \in (0, 1)$, respectively. Thus, the manager’s perceived probability of success is $p_m(e) = \theta_m e$ and the agent’s perceived probability of success is $p_w(e) = \theta_w e$. Alternatively, $\theta_m$ and $\theta_w$ can be interpreted as each party’s subjective beliefs about the worker’s ability (marginal productivity of effort).

A key element of our model is that the manager and the worker have differing prior beliefs. They openly disagree about the working environment or the worker’s ability. As stressed in the Introduction, they do not update their beliefs when they are confronted with someone who holds different beliefs. This implies that each player believes that he is right and others are wrong. In addition, each player is aware that other players believe the opposite. If this is not the case, they update their beliefs when meeting someone with different beliefs, and so reach agreement. That is, our model is reduced to the standard moral hazard model with a common

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Footnotes:

3 As Harsanyi (1967) wrote “by the very nature of subjective probabilities, even if two individuals have exactly the same information and are at exactly the same high level of intelligence, they may very well assign different subjective probabilities to the very same events.”
prior ($\theta_m = \theta_w$).

We allow $\theta_m$ to be greater than or less than $\theta_w$. When $\theta_m > \theta_w$, the manager is more optimistic about the success of the task than the worker. When $\theta_m < \theta_w$, the manager is less optimistic. Note that the workers share the symmetric beliefs. This assumption reflects the fact that the workers are in the same situation and are more likely to build cognitive biases in the same direction compared to the manager. In Section 4.3, however, we extend the model to account for the case in which the workers are allowed to have heterogeneous beliefs.

The manager offers an incentive contract contingent on the outcome of the task. Thus, when only one agent is considered, the contract stipulates two different transfers, $v^1 \equiv (v^S, v^F) \in \mathbb{R}^2$. The worker receives $v^S$ in the case of $x = S$ and $v^F$ in the case of $x = F$. The risk averse worker’s expected payoff is:

$$U(e) = p_w(e)u(v^S) + (1 - p_w(e))u(v^F) - ce.$$  \hspace{1cm} (1)

with $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u(0) = 0$ and $u'(0) = \infty$.

When the manager wants to hire two workers, $i = 1, 2$, the manager can confront four different situations and offers a contract $v^2 \equiv (v^{SS}, v^{SF}, v^{FS}, v^{FF}) \in \mathbb{R}^4$, where $v^{x;i}$ represents a transfer given to worker $i$ in each situation, $x_i \in \{S, F\}$ and $x_j \in \{S, F\}$. Both workers succeed with probability $p_w(e_1)p_w(e_2)$, worker 1 succeeds but worker 2 fails with $p_w(e_1)(1 - p_w(e_2))$, worker 2 succeeds but worker 1 fails with $(1 - p_w(e_1))p_w(e_2)$, and neither worker succeeds with $(1 - p_w(e_1))(1 - p_w(e_2))$. We focus on the symmetric case. Worker $i$’s expected payoff is:

$$U_i(e_i, e_j) = p_w(e_i)p_w(e_j)u(v^{SS}) + p_w(e_i)(1 - p_w(e_j))u(v^{SF})$$
$$+ (1 - p_w(e_i))p_w(e_j)u(v^{FS}) + (1 - p_w(e_i))(1 - p_w(e_j))u(v^{FF}) - ce_i$$ \hspace{1cm} (2)

Following the literature, we will refer to $v^2$ as joint performance evaluation if $(v^{SS}, v^{FS}) > (v^{SF}, v^{FF})$ and as relative performance evaluation if $(v^{SS}, v^{FS}) < (v^{SF}, v^{FF})$. The inequality indicates weak inequality of each component and strict inequality for at least one component. A worker’s effort generates positive externalities to his partner under JPE, while negative externalities under RPE. For instance, when a worker succeeds in the task, his compensation is either $v^{SS}$ or $v^{SF}$ depending on whether his partner succeeds or fails. Hence, when $v^{SS} > v^{SF}$, the partner’s good performance makes the worker better off. On the other hand, when $(v^{SS}, v^{FS}) = (v^{SF}, v^{FF})$, it can be easily seen that $U_i(e_i, e_j)$ is reduced to $U(e)$. That is, the compensation scheme is individual performance evaluation (hereafter IPE).

Finally, the manager’s payoff is revenue from the tasks net any payments made to the workers. The task yields revenue $R^S$ if it succeeds, and revenue $R^F$ if it fails, where $\Delta R \equiv R^S - R^F$. As in Grossman and Hart (1983), we will consider a two-stage problem – (i) the manager finds the best wage scheme which minimizes the expected payment to the workers, given any effort level, and (ii) the manager chooses the optimal level of effort which maximizes the expected
payoff. In the beginning, we assume that \( \Delta R \) is large enough. Hence, the manager always induces the workers to supply effort so that we can focus on the cost minimization problem – the manager’s objective is minimizing the expected payment to the workers based on her beliefs, \( \theta_m \). We extend the analysis later to allow for the effort selection problem in Section 4.1.

The timing of the game is as follows. In the first stage, a manager offers a contract to worker(s), who can accept or reject it. When two workers are considered, if one of the workers rejects the offer, then the game is reduced to the one-worker case. In the second stage, if the contract is accepted by both, the workers decide whether to work or not simultaneously. As a result of effort, in the third stage, the outcome of the task is realized and the worker(s) receives payments as stipulated in the contract.

3 Incentive policies and organization design

3.1 One worker

As a benchmark, we first consider the case with one worker.\(^4\) The manager’s perceived expected payment is \( p_m(e)v^S + (1 - p_m(e))v^F \). When the manager induces the worker’s effort \( e = 1 \), the minimum payment contract solves:

\[
\min \Psi^1 \equiv \theta_m v^S + (1 - \theta_m)v^F
\]

subject to

\[\theta_w(u(v^S) - u(v^F)) \geq c\] \hspace{1cm} (3)

\[\theta_w u(v^S) + (1 - \theta_w)u(v^F) - c \geq \pi\] \hspace{1cm} (4)

The first constraint (3) is the incentive compatibility constraint that requires the worker to exert effort, \( U(e = 1) \geq U(e = 0) \). As usual, if the worker is indifferent between work and shirk, we assume that he prefers to work as a tie-breaking rule. It is intuitive that the higher the worker’s beliefs, the less stringent the constraint is. That is, lower-powered incentives \( u(v^S) - u(v^F) \) can induce the worker’s effort. The second constraint (4) is the participation constraint which ensures that the worker is at least as well off with the contract as with the outside option, \( U(e = 1) \geq \pi \). The following proposition characterizes the optimal contract.

**Proposition 1** The optimal contract implementing effort entails:

- There exists \( \bar{\theta}_m \) such that for every \( \theta_m \leq \bar{\theta}_m \), \( v^S \) and \( v^F \) are characterized by \( \frac{u'(v^F)}{u'(v^S)} = \frac{\theta_w/(1-\theta_w)}{\theta_m/(1-\theta_m)} \) and \( \theta_w u(v^S) + (1 - \theta_w)u(v^F) = c + \pi \), and for every \( \theta_m > \bar{\theta}_m \), \( u(v^S) = \frac{\pi}{\theta_w} + \pi \) and \( u(v^F) = \pi \).

\(^4\)Most papers look at the first-best outcome as a benchmark case; however, it is not our main interest. We want to compare the one-agent case and the two-agent case in the presence of moral hazard. One can refer to Santos-Pinto (2008) and de la Rosa (2012) to see the first-best outcome in a more general setup.
Proof. In the appendix. ■

(Figure 1. Optimal Contract)

When $\theta_m \leq \bar{\theta}_m$, only the participation constraint (4) binds. As in Figure 1-(a), the optimal compensation can be found by comparing the slope of the isocost line and that of the participation constraint:

$$\frac{1 - \theta_m}{\theta_m} = \frac{(1 - \theta_w) u'(v_F)}{\theta_w u'(v_S)} = -\frac{dv^S}{dv^F}_{U(e=1)=\pi}. \quad (5)$$

The isocost line is the locus of wage combinations along which the manager’s perceived expected payment is constant. The LHS of (5) is the slope of the isocost line, whereas the RHS of (5) is the slope of the participation constraint. The slope of the isocost line describes the manager’s perceived relative costs of success and failure-contingent wages according to her own beliefs. The slope of the participation cost is the worker’s perceived relative benefits of two types of wages.

In equilibrium, (5) demonstrates that the manager’s perceived relative costs of $v^S$ and $v^F$ are equal to the worker’s perceived relative benefits of those. As the manager believes the success of the task is less likely, she thinks that the expected payment is lower by offering more of the success-contingent wage and less of the failure-contingent wage. This is the reason why the incentive constraint is not binding. In addition, the manager with a more pessimistic outlook is more willing to provide higher-powered incentives, i.e., $v^S/v^F$ is decreasing in $\theta_m$.

When $\theta_m$ becomes large, the incentive constraint (3) begins to bind as well. The optimal compensation can be readily found by solving the two constraints. The optimal wage no longer depends on the manager’s beliefs. In other words, when the manager is relatively optimistic and contracts with a single worker, the manager’s beliefs do not affect the incentive policies in organization.

3.2 Two workers

We now turn to the case with two workers where the manager is able to offer an interdependent wage scheme. The manager’s perceived expected payment to worker $i = 1, 2$ is $p_m(e_i)p_m(e_j)v^{SS} + p_m(e_i)(1 - p_m(e_i))v^{SF} + (1 - p_m(e_i))p_m(e_j)v^{FS} + (1 - p_m(e_i))(1 - p_m(e_j))v^{FF}$. When the manager implements $e = 1$, the minimum payment contract for each worker solves:

$$\min \Psi^2 \equiv \theta^2_m v^{SS} + \theta_m (1 - \theta_m) \left( v^{SF} + v^{FS} \right) + (1 - \theta_m)^2 v^{FF}$$
subject to

\[ \theta_w^2(u(v^{SS}) - u(v^{FS})) + \theta_w(1 - \theta_w)(u(v^{SF}) - u(v^{FF})) \geq c \]  

(6)

\[ \theta_w^2 u(v^{SS}) + \theta_w(1 - \theta_w) u(v^{SF}) + \theta_w(1 - \theta_w) u(v^{FS}) + (1 - \theta_w)^2 u(v^{FF}) - c \geq \bar{u} \]  

(7)

As in the one-worker case, the manager minimizes the expected payment to the workers with two constraints: an incentive compatibility constraint and a participation constraint. The incentive compatibility constraint (6) is derived from the Nash equilibrium condition i.e., \( U_i(e_i = e_j = 1) \geq U_i(e_i = 0, e_j = 1) \) for \( i = 1, 2 \) and \( i \neq j \). That is, in a Nash equilibrium, the wages are provided enough for both workers to exert effort.\(^5\) Also, the participation constraint (7) ensures each worker’s acceptance of the contract, i.e., \( U_i(e_i = e_j = 1) \geq \pi_i \). The following proposition characterizes the optimal contract.

**Proposition 2** The optimal contract implementing effort entails:

- For every \( \theta_m \) and \( \theta_w \), \( v^{SS}, v^{SF}, v^{FS}, \) and \( v^{FF} \) are characterized by

\[
\frac{1 - \theta_m}{\theta_m} = \frac{(1 - \theta_w) u'(v^{SF})}{u'(v^{SS})} \quad \text{(8)}
\]

\[
= \frac{(1 - \theta_w) u'(v^{FF})}{u'(v^{FS})} \quad \text{(9)}
\]

\[ \theta_w u(v^{SS}) + (1 - \theta_w) u(v^{SF}) = \bar{u} + \frac{c}{\theta_w}, \text{ and} \]

\[ \theta_w u(v^{FS}) + (1 - \theta_w) u(v^{FF}) = \bar{u}. \quad \text{(10)} \]

**Proof.** In the appendix. \( \blacksquare \)

Since both (6) and (7) bind, they can be solved together and compactly rewritten as (10) and (11). Indeed, the manager faces the two separate problems. She chooses the optimal combination of \( v^{SS} \) and \( v^{SF} \) to satisfy the incentive compatibility constraint. She also chooses the optimal combination of \( v^{FS} \) and \( v^{FF} \) to meet the participation constraint. As illustrated in Figure 1-(b), when we set the slope of the iso-cost line to the slope of each constraint,

\[
\left. \frac{dv^{SS}}{dv^{SF}} \right|_{U_i(e_i = e_j = 1), U_i(e_i = 0, e_j = 1)} = \left. \frac{dv^{FS}}{dv^{FF}} \right|_{U_i(e_i = e_j = 1) = \pi_i}
\]

we obtain (8) and (9).

\(^5\)This incentive compatibility constraint does not exclude another Nash equilibrium in which both workers shirk. In an earlier version of the paper, we studied the case where the supply of effort is the dominant-strategy. The qualitative results are not changed. That is, the contract is robust to manipulative coordination by the workers.
Corollary 3 The optimal contract shows:

- When $\theta_m > \theta_w$, the RPE scheme ($v^{SS} < v^{SF}$ and $v^{FS} < v^{FF}$) is preferred.
- When $\theta_m = \theta_w$, the IPE scheme ($v^{SS} = v^{SF}$ and $v^{FS} = v^{FF}$) is preferred.
- When $\theta_m < \theta_w$, the JPE scheme ($v^{SS} > v^{SF}$ and $v^{FS} > v^{FF}$) is preferred.

In equilibrium, (8) and (9) demonstrate that the manager’s perceived relative costs of $v^{SS}(v^{FF})$ and $v^{SF}(v^{FS})$ are equal to the worker’s perceived relative benefits of those. When there is no disagreement among the contracting parties, i.e., $\theta_m = \theta_w$, the optimal compensation scheme is individual performance evaluation. When the manager shares the common beliefs with the workers, the manager does not offer an interdependent compensation structure because it increases each worker’s risk exposure. However, the disagreement changes the optimal compensation scheme in a significant way.

When the manager is more optimistic ($\theta_m > \theta_w$), her choice of incentive scheme is RPE. The workers believe that they will receive $v^{SF}$ ($v^{FF}$) relative to $v^{SS}$ ($v^{FS}$) more frequently vis-à-vis the manager. The reason is that the workers underestimate the probability of success in the manager’s point of view. Knowing this fact, the manager finds it less costly to provide RPE. On the other hand, when the manager underestimates the workers’ abilities relative to the workers’ own beliefs ($\theta_m < \theta_w$), the workers expect to receive $v^{SS}$ ($v^{FS}$) relative to $v^{SF}$ ($v^{FF}$) more often than the manager expects to pay. Hence, JPE is less costly to the manager.

These results show how organizational incentive structures can be shaped by managers’ beliefs and their disagreement with their employees. When CEOs or managers are more optimistic relative to employees, our model predicts that they are more likely to promote internal competition to incentivize workers. By contrast, when managers are more pessimistic, organizations may provide team-based rewards.

3.3 Allocation of workers

We examine how the manager wants to allocate one or two workers among tasks depending on the manager’s and the workers’ beliefs. For this, we consider two types of managers, $\theta_m \in \{\theta_l, \theta_h\}$, and suppose there is a continuum of workers who hold different beliefs $\theta_w \in [\theta_l, \theta_h]$ for a task. When $\theta_m = \theta_l$, the manager can contract with workers who hold stronger beliefs,

6 Relating to this point, it is well-known in the literature that RPE is optimal when there exists a common shock affecting both workers’ performance. In this case, RPE reduces each worker’s risk exposure by filtering out the common shock.

7 Alternatively, one may consider a situation in which workers have the choice between multiple managers as in Van den Steen (2005). However, in our model, the participation constraint is binding, and thus the workers’ equilibrium payoffs are $\pi$ regardless of their beliefs. This is the reason why we study the job assignment problem for sorting in the labor market.
whereas when $\theta_{m} = \theta_{h}$, the manager can contract with workers who hold weaker beliefs. For analytical simplicity, we assume that $\theta_{i}$ is not too small.

When the manager interacts with a single worker, not surprisingly, the right person for the task is the worker who holds the strongest beliefs $\theta_{w} = \theta_{h}$. This result is well established in the literature and can be understood by investigating how the manager’s expected payment changes with the workers’ beliefs. With a single worker, it can be shown that the manager’s perceived expected payment is decreasing with the worker’s beliefs:

$$\frac{\partial \Psi_{1}}{\partial \theta_{w}} = -(\lambda_{1} + \lambda_{2})(u(\bar{v}^{S}) - u(\bar{v}^{F})) < 0.$$ (12)

This is obtained by applying the envelope theorem to the manager’s minimization problem. $\lambda_{1}$ and $\lambda_{2}$ are the Lagrange multipliers attached to the incentive compatibility constraint (3) and the participation constraint (4) respectively. Whether each constraint is binding or not, the manager’s expected payment is decreasing in $\theta_{w}$, because the manager can save a little bit of $u(\bar{v}^{S}) - u(\bar{v}^{F})$. It immediately implies that the manager prefers to assign the worker with the strongest beliefs to a task regardless of her own beliefs.

However, when the manager interacts with two workers, the manager’s decision for the allocation of workers is not straightforward. The manager’s expected payment is not monotonic with the workers’ beliefs. This can be understood in the following:

$$\frac{\partial \Psi_{2}}{\partial \theta_{w}} = -\lambda_{1} \left[ u(\bar{v}^{SS}) - u(\bar{v}^{SF}) + \frac{c}{\theta_{w}^{2}} \right] - \lambda_{2} \left[ u(\bar{v}^{FS}) - u(\bar{v}^{FF}) \right].$$ (13)

Again, $\lambda_{1}$ and $\lambda_{2}$ are the Lagrange multipliers associated with the incentive compatibility constraint (10) and the participation constraint (11). The sign of the above is determined by whether the wage scheme is JPE or RPE. Under JPE, i.e., $(\bar{v}^{SS}, \bar{v}^{FS}) > (\bar{v}^{SF}, \bar{v}^{FF})$, the expected payment is decreasing in $\theta_{w}$. On the other hand, under RPE, i.e., $(\bar{v}^{SS}, \bar{v}^{FS}) < (\bar{v}^{SF}, \bar{v}^{FF})$, the expected payment is increasing in $\theta_{w}$ unless it is not too small. Thus, the both types of bias can be exploited by the manager.

The intuition is as follows. Recall that the manager dealing with two workers is now able to create two types of incentives – one for joint performance and another for relative performance. Disagreement creates a discrepancy between the manager’s relative price of the two incentives and the workers’ relative responsiveness to the two incentives. The discrepancy allows the manager to optimally mix the two incentives at a lower price. Suppose $\theta_{m} = \theta_{w}$. The manager’s optimal contract is IPE, $\bar{v}^{SS} = \bar{v}^{SF}$ and $\bar{v}^{FS} = \bar{v}^{FF}$. When $\theta_{w}$ becomes slightly lower than $\theta_{m}$, the manager can save a little bit of payment by reducing $\bar{v}^{SS}$ and $\bar{v}^{FS}$, and then offering RPE from her own perspective.

The discussion implies that the manager’s expected payment is hump-shaped in the workers’ beliefs $\theta_{w}$ with a maximum at $\theta_{w} = \theta_{m}$. Thus, the manager wants to assign the workers
with as different beliefs from herself as possible. Namely, when $\theta_m = \theta_l$, the manager allocates the most optimistic workers to the tasks. When $\theta_m = \theta_h$, the manager allocates the least optimistic workers to the tasks.

**Proposition 4** When there is a continuum of workers who hold different beliefs $\theta_w \in [\theta_l, \theta_h]$ for a task:

- In the one-worker case, the manager wants to assign the worker with the strongest beliefs $\theta_w = \theta_h$ regardless of $\theta_m \in \{\theta_l, \theta_h\}$.
- In the two-worker case, the manager with $\theta_m = \theta_l$ wants to assign the workers with the strongest beliefs $\theta_w = \theta_h$, while the manager with $\theta_m = \theta_h$ prefers to assign the workers with the weakest beliefs $\theta_w = \theta_l$.

**Proof.** In the appendix.

We can consider a continuum of managers, $\theta_m \in [\theta_l, \theta_h]$. Intuitively, a manager’s selection depends on how close $\theta_m$ to $\theta_h$ relative to $\theta_l$. When a manager’s beliefs about a task is relatively lower, she seeks to match with workers who have strong confidence. When a manager is rather optimistic about a task, she prefers to match with workers with weak confidence instead.

The sorting outcome in our model is different from Van den Steen (2005) which shows that the beliefs of a manager and employees are more aligned. Unlike Van den Steen (2005), our model predicts heterogeneous matching. The principal can use the monetary incentives to take advantage of disagreement in our model, whereas the agency problem is absent from his model. In some sense, the difference is obvious and intuitive. When disagreement can be handled in an advantageous way, a heterogeneous matching will prevail. On the other hand, as in Van den Steen (2005), when agents are willing to work harder for firms that espouse a vision they agree with, a homogeneous matching will occur.

**Example.** Suppose $\bar{u} = 0$ and $u(v) = v$. The workers’ risk neutrality allows us to find the principal’s expected payments to the workers in a closed form. As $u(v) = v$, both the isocost line and the incentive compatibility constraint are linear, so that we must have a corner solution with limited liability $v^2 \geq 0$. With $\theta_l < \theta_w$, the JPE scheme is $v^{SS} = c/\theta_w^2$ and $v^{SF} = v^{FS} = v^{FF} = 0$. The manager’s expected payment is $\Psi^2 = [\theta_m/\theta_w]^2 c$. On the other hand, with $\theta_m > \theta_w$, the RPE scheme is $v^{SF} = c/\theta_w(1 - \theta_w)$ and $v^{SS} = v^{FS} = v^{FF} = 0$. The manager’s expected payment is $\Psi^2 = [\theta_m(1 - \theta_m)/\theta_w(1 - \theta_w)] c$. The manager’s expected payment with JPE is decreasing in $\theta_w$, while with RPE it is increasing in $\theta_w$ as long as $\theta_w \geq 1/2$. In this case, there exist $\hat{\theta}_m = \theta_h^2 / (\theta_h^2 + \theta_l(1 - \theta_l))$ such that for every $\theta_m < \hat{\theta}_m$, the manager wants to assign $\theta_w = \theta_h$, while for every $\theta_m > \hat{\theta}_m$, the manager prefers to assign $\theta_w = \theta_l$. 

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4 Extensions

4.1 Effort selection

Here we investigate the manager’s effort selection problem to compete the analysis. We now assume that $\Delta R$ is not sufficiently large:

$$\Delta R < \frac{c}{\theta_w} + \frac{\bar{w} - u^{-1}(\bar{w})}{\theta_m}. \quad (14)$$

The manager may choose not to induce effort depending on parameter values. In particular, we are interested in how the manager’s decision varies with the workers’ beliefs.

We restrict attention to a symmetric equilibrium, and then the optimal contract solves

$$\max \ V(\theta) = p_m(e) R^S + (1 - p_m(e)) R^F$$

$$- p_m(e)^2 w^S + p_m(e) (1 - p_m(e)) (w^S + w^F) + (1 - p_m(e))^2 v^{FF}$$

subject to

$$e \in \arg\max_{\tilde{e}} U_i(\tilde{e}, e) \quad (15)$$

$$U_i(e, e) \geq \bar{\pi} \quad (16)$$

We have already analyzed the case in which effort is implemented. The manager’s expected payoff for each worker is

$$V(e = 1) = \theta_m R^S + (1 - \theta_m) R^F - \Psi^2. \quad (17)$$

The implementation of no effort is trivial. The contractual outcome simply requires the workers’ utility to be the reservation wage: $v^S = 0$ and $v^F = u^{-1}(\bar{w})$ in the one-worker case, and $v^{SS} = v^{SF} = v^{FS} = 0$ and $v^{FF} = u^{-1}(\bar{w})$ in the two-worker case.\textsuperscript{8} In both cases, the manager’s expected payoff for each worker is

$$V(e = 0) = R^F - u^{-1}(\bar{w}). \quad (18)$$

Comparing the two (17) and (18), we get the following results.

**Proposition 5** The manager’s decision to induce effort is as follows. There exist $\theta_w$ and $\bar{\theta}_w$ such that

- for every $\theta_w > \bar{\theta}_w$ and $\theta_w < \bar{\theta}_w$, the manager induces $e = 1$, and

\textsuperscript{8}The one-worker case can be obtained by imposing additional constraints such as $v^S = v^{SS} = v^{SF}$ and $v^F = v^{FS} = v^{FF}$. In this case, there exists $\bar{\theta} < 1$, where $\bar{\theta} = (\Delta R - \Psi) \frac{\theta}{\phi}$, such that when $\theta_w > \bar{\theta}$, the manager induces $e = 1$. When $\theta_w < \bar{\theta}$, the manager induces $e = 0$. 
• for every $\theta_w \in [\theta_w, \overline{\theta}_w]$, the manager induces $e = 0$.

Proof. In the appendix.

Perhaps surprisingly, the manager chooses to induce effort from the workers not only with relatively strong beliefs but with relatively weak beliefs. But effort is not induced when the workers’ beliefs are intermediate. In fact, these results are immediate from the fact that the manager’s expected payment is hump-shaped in the workers’ beliefs, as in (13).

4.2 Continuous effort

We now want to show that our main results are robust to continuous effort, $e \in [0, 1]$. We now consider a strictly convex cost function $C(e)$ with $\lim_{e \to 1} C(e) = \infty$, and hence the second-order conditions are satisfied. Each worker maximizes his utility, and we get the best response function as

$$e_i(e_j) \in \arg\max_{\hat{e}_i} U_i(\hat{e}_i, e_j).$$

Given the incentive structure, the symmetric Nash equilibrium $e^* = e_i = e_j$ is characterized by

$$\theta_w e^* \left[ u(\tilde{v}^{SS}) - u(\tilde{v}^{FS}) \right] + \theta_w (1 - \theta_w e^*) \left[ u(\tilde{v}^{SF}) - u(\tilde{v}^{FF}) \right] = C'(e^*),$$

which becomes the incentive compatibility constraint for the manager’s contracting problem. The participation constraint is written as

$$\theta_w e^* u(\tilde{v}^{SS}) + \theta_w e^* (1 - \theta_w e^*) \left( u(\tilde{v}^{SF}) + u(\tilde{v}^{FS}) \right) + (1 - \theta_w e^*)^2 u(\tilde{v}^{FF}) - C(e^*) \geq \overline{u}.$$

Two constraints can be solved together, and we obtain

$$\theta_w e^* u(\tilde{v}^{SS}) + (1 - \theta_w e^*) u(\tilde{v}^{SF}) = \overline{u} + C(e^*) - C'(e^*) e^* + \frac{C'(e^*)}{\theta_w} \quad (19)$$

$$\theta_w e^* u(\tilde{v}^{FS}) + (1 - \theta_w e^*) u(\tilde{v}^{FF}) = \overline{u} + C(e^*) - C'(e^*) e^*. \quad (20)$$

Note that (19) and (20) are equivalent to (6) and (7) in the case of binary effort. The incentives for effort provision are given by the optimal combination of $\tilde{v}^{SS}$ and $\tilde{v}^{SF}$, and the participation is guaranteed by the optimal combination of $\tilde{v}^{FS}$ and $\tilde{v}^{FF}$. One can easily see that the rest of the analysis is no different from previous one. When the manager wants to induce any given effort $e^*$ from the workers, the minimum payment contract has to set the manager’s perceived relative costs equal to the workers’ perceived relative benefits of $\tilde{v}^{SF}$ ($\tilde{v}^{FF}$) and $\tilde{v}^{SS}$ ($\tilde{v}^{FS}$).

**Proposition 6** When effort is continuous, the optimal contract entails:
• Given any effort $e^*$, as $\theta_m \geq \theta_w$, we obtain $v^{SS} \leq v^{SF}$ and $v^{FS} \leq v^{FF}$.

• $v^{SS}/v^{SF}$ and $v^{FS}/v^{FF}$ are decreasing with $e^*$ with $\theta_m > \theta_w$, while $v^{SS}/v^{SF}$ and $v^{FS}/v^{FF}$ are increasing with $e^*$ with $\theta_m < \theta_w$.

The results about the manager’s choice of incentive structures do not change in the continuous effort case. But it is hard to analyze the job assignment problem because the manager wants to implement different levels of effort depending on the workers’ beliefs. We can show, on the other hand, that the intensity of RPE and JPE are increasing with the effort level that the manager wants to implement.

4.3 Heterogeneous beliefs

We now consider the case where the workers have asymmetric beliefs from each other. In this case, it is more natural to interpret $\theta_w$ as the worker’s ability. As in Santos-pinto (2008), worker $i = 1, 2$ has beliefs about his own ability but has another beliefs about worker $j$’s ability. Let us denote by $\theta_{ij}$ worker $i$’s beliefs about his own ability, and by $\theta_{ji}$ worker’s $i$’s beliefs about worker $j$’s ability. The premise “agree to disagree” applies not only between the manager and the workers but also between the two workers.

When this information is observed by the manager, she can offer different wage $v^2_i$ and $v^2_j$. The minimum payment contract to worker $i$ has to satisfy the two constraints, which are shown to be reduced to

$$\theta_{ij} u(v^{SS}_i) + (1 - \theta_{ij})u(v^{SF}_i) \geq \pi + \frac{c}{\theta_{ij}} \quad (21)$$

$$\theta_{ji} u(v^{FS}_i) + (1 - \theta_{ji})u(v^{FF}_i) = \pi. \quad (22)$$

From (21) and (22), we can see that the choice of JPE or RPE is determined by each worker’s beliefs about the other worker’s ability relative to the manager’s beliefs. That is, when the manager offers a contract to worker $i$, whether JPE or RPE is offered depends not on worker $i$’s beliefs about his own ability, but on his beliefs about worker $j$’s ability. This leads to a possibility that the manager offers the JPE wage scheme to one worker, while the RPE wage scheme to another worker. For example, when $\theta_{ij} < \theta_m < \theta_{ji}$, the incentive structures $v^2_i$ for worker $i$ follows RPE, and $v^2_j$ worker $j$ follows JPE. Note that $\theta_{ij}$ is in the RHS of the incentive compatibility constraint (21). That is, although the worker’s beliefs about his own ability do not affect whether the incentive scheme is JPE or RPE, they affect the intensity of incentives.

Proposition 7 When the workers have heterogenous beliefs,

• The optimal contract $v^2_i$ offered to worker $i$ follows RPE and JPE as $\theta_m$ is greater or smaller than $\theta_{ij}$, where $i = 1, 2$, and $i \neq j$, respectively.
Consider the allocation problem of workers. When there is a continuum of workers who hold different beliefs about his co-worker \( \theta_i \in [\theta_l, \theta_h] \) for a task, where \( i = 1, 2, \) and \( i \neq j \), the manager with \( \theta_m = \theta_i \) wants to assign the workers with the strongest beliefs about each other \( \theta_i = \theta_j = \theta_h \). The manager with \( \theta_m = \theta_h \) wants to assign the workers with the weakest beliefs about each other \( \theta_i = \theta_j = \theta_l \).

Santos-Pinto (2008) also looks at the case that the agents hold mistaken beliefs about their co-workers while they have correct beliefs about their own abilities. In this setup, he shows that the principal prefers to use an interdependent incentive scheme. Compared to this, we are now able to show that the manager determines the incentive scheme between RPE or JPE depending on whether her beliefs are greater or smaller than each worker’s beliefs about the other worker’s ability.

5 Concluding remarks

This article studies an optimal contract design between a manager and multiple workers who have different beliefs about working environments. A manager and workers evaluate contracts according to their own beliefs, and hence a manager finds it worthwhile to offer an interdependent incentive structure. This article provides a new rationale for the adoption of joint performance evaluation and relative performance evaluation as a result of the difference between managers’ and workers’ beliefs. We have further studied how a manager wants to assign workers to jobs and show that disagreement emerges from managerial decisions.

There are several new empirical implications of our model that could serve as guides for future empirical or experimental tests about managers’ choices of incentive structures. First, when a manager is more (less) optimistic than workers, the optimal incentive structures are relative (joint) performance evaluation. Second, a manager with strong beliefs is more likely to match with workers with weak beliefs, and vice versa. Managers’ optimism is proxied by their option exercise behavior in the literature of corporate finance. Quantitative measures of RPE or JPE are not difficult to construct. For example, the magnitude of RPE can be measured by the use of up-or-out promotion and forced ranking system, and that of JPE by the use of profit sharing.

Previous empirical works have focused primarily on executive compensation for a CEO as an agent in the principal-agent model when CEO’s optimism exists. We believe it is also important how a CEO as a principal designs incentive structures for organizations according to her optimism. In this line of research, Bertrand and Schoar (2003) find that manager fixed effects matter in the determination of corporate practices such as investment policy, financial policy, R&D expenditures, advertising expenditures and so on. Malmendier, Tate, and Yan (2011) also study the effect of managers’ optimism about their firms’ future cash flows on equity.
financing. However, we are unaware of any empirical studies investigating the principal’s incentive provision according to her beliefs.

Our model also suggests caution in the empirical analysis for the relationship between the workers’ confidences and the intensity of incentives. Proposition 5 shows that the manager’s effort selection is non-monotonic with the workers’ beliefs. In the binary effort case, the manager does not induce effort from the workers when her beliefs are not much different from the workers’. This result implies that the manager’s provision of incentives could be non-monotonic with the workers’ confidences.

6 Appendix

Proof of Proposition 1.

The Lagrangian of the problem is:

\[
\mathcal{L} = \theta_m v^S + (1 - \theta_m) v^F - \lambda_1 \left[ \theta_w (u(v^S) - u(v^F)) - c \right] - \lambda_2 \left[ \theta_w u(v^S) + (1 - \theta_w) u(v^F) - c - \bar{u} \right].
\] (23)

The first-order conditions are:

\[
\frac{\partial \mathcal{L}}{\partial v^S} = \theta_m - (\lambda_1 + \lambda_2) \theta_w u'(v^S) = 0,
\] (24)

\[
\frac{\partial \mathcal{L}}{\partial v^F} = (1 - \theta_m) + (\lambda_1 \theta_w - \lambda_2 (1 - \theta_w)) u'(v^F) = 0,
\] (25)

\[
\frac{\partial \mathcal{L}}{\partial \lambda_1} = \theta_w (u(v^S) - u(v^F)) - c \geq 0, \quad \lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0,
\] (26)

\[
\frac{\partial \mathcal{L}}{\partial \lambda_2} = \theta_w u'(v^S) + (1 - \theta_w) u(v^F) - c - \bar{u} \geq 0, \quad \lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = 0.
\] (27)

When \( \theta_w \) is large enough and thus \( \lambda_1 = 0 \), (24) and (25) yield

\[
\frac{u'(v^F)}{u'(v^S)} = \frac{\theta_w / (1 - \theta_w)}{\theta_m / (1 - \theta_m)},
\]

which characterizes the solution together with the participation constraint. Thus, we must have \( v^S > v^F > 0 \). However, as a fall in \( \theta_w \) makes the incentive compatibility constraint bind, we have \( \lambda_1 > 0 \). In this case, solving (26) and (27) together, we get \( u(v^S) = \frac{c}{\theta_w} + \bar{u} \) and
\[ u(v^F) = \bar{u}. \] Hence, the cutoff \( \tilde{\theta}_m \) is characterized by

\[
\frac{u'(\bar{u})}{u'(c/\theta_w + \bar{u})} = \frac{\theta_w/(1 - \theta_w)}{\theta_m/(1 - \theta_m)}.
\]

Since the LHS is greater than 1 due to \( u'(\cdot) < 0 \), we must have \( \tilde{\theta}_m < \theta_w. \]

**Proof of Proposition 2 and Corollary 3.**

Using (10) and (11), the Lagrangian of the problem is written as

\[
L = \theta_m^2 v^{SS} + \theta_m(1 - \theta_m)v^{SF} + \theta_m(1 - \theta_m)v^{FS} + (1 - \theta_m)^2 v^{FF} - \lambda_1 \left[ \theta_w u(v^{SS}) + (1 - \theta_w)u(v^{SF}) - \bar{u} - \frac{c}{\theta_w} \right] - \lambda_2 \left[ \theta_w u(v^{FS}) + (1 - \theta_w)u(v^{FF}) - \bar{u} \right]
\]

The first-order conditions are:

\[
\frac{\partial L}{\partial v^{SS}} = \theta_m - \lambda_1 \theta_w u'(v^{SS}) = 0, \quad (29)
\]

\[
\frac{\partial L}{\partial v^{SF}} = \theta_m(1 - \theta_m) - \lambda_1 (1 - \theta_w) u'(v^{SF}) = 0, \quad (30)
\]

\[
\frac{\partial L}{\partial v^{FS}} = \theta_m(1 - \theta_m) - \lambda_2 \theta_w u'(v^{FS}) = 0, \quad (31)
\]

\[
\frac{\partial L}{\partial v^{FF}} = (1 - \theta_m)^2 - \lambda_2 (1 - \theta_w) u'(v^{FF}) = 0, \quad (32)
\]

\[
\frac{\partial L}{\partial \lambda_1} = \theta_w u(v^{SS}) + (1 - \theta_w)u(v^{SF}) - \bar{u} - \frac{c}{\theta_w} = 0, \quad (33)
\]

\[
\frac{\partial L}{\partial \lambda_2} = \theta_w u(v^{FS}) + (1 - \theta_w)u(v^{FF}) - \bar{u} = 0. \quad (34)
\]

Note that the choice of \( v^{SS} \) and \( v^{SF} \) and that of \( v^{FS} \) and \( v^{FF} \) are separable. (29) and (30) can be solved together, and we obtain (8). Likewise, (31) and (32) can be solved together, and we obtain (9). Thus, as \( \theta_m \gtrsim \theta_w \), we must have \( u'(v^{SS}) \gtrsim u'(v^{SF}) \) and \( u'(v^{FS}) \gtrsim u'(v^{FF}) \). Since \( u'(\cdot) < 0 \), we can conclude that as \( \theta_m \gtrsim \theta_w \), \( v^{SS} \lesssim v^{SF} \) and \( v^{FS} \lesssim v^{FF} \). The exact solutions of equilibrium wages can be characterized by solving (8), (9), (33) and (34) together.
Proof of Proposition 4.

Let us first show that the effect of a change in $\theta_w$ on the manager’s expected payment. In the one-worker case, by applying the envelope theorem to (23), we have

$$\frac{\partial L}{\partial \theta_w} = \frac{\partial \Psi^1}{\partial \theta_w} = -(\lambda_1 + \lambda_2)(u(v^S) - u(v^F)) < 0.$$  

Thus, the manager’s expected payment to the worker is decreasing in $\theta_w$. Thus, when there is a continuum of workers $\theta_w \in [\theta_l, \theta_h]$ for tasks, the manager always prefers to assign those with the strongest beliefs to the tasks.

In the two-worker case, likewise, we apply the envelope theorem to (28), and get

$$\frac{\partial L}{\partial \theta_w} = \frac{\partial \Psi^2}{\partial \theta_w} = -\lambda_1 \left[ u(v^{SS}) - u(v^{SF}) + c/\theta_w^2 \right] - \lambda_2 \left[ u(v^{FS}) - u(v^{FF}) \right].$$

Under JPE ($v^{SS} > v^{SF}$ and $v^{FS} > v^{FF}$), the manager’s expected payment is decreasing in $\theta_w$. The analysis, however, becomes a bit complicated when the optimal wage scheme follows RPE ($v^{SS} < v^{SF}$ and $v^{FS} < v^{FF}$). When $\theta_w$ is small enough, we cannot determine the sign of $\frac{\partial \Psi^2}{\partial \theta_w}$. Since $v^{SS}/v^{SF}$ and $v^{FS}/v^{FF}$ is decreasing in $\theta_w$ from (23) and (28), there must be a cutoff $\theta_w$ above which $\frac{\partial \Psi^2}{\partial \theta_w} > 0$. When $\theta_l$ is not too small, we can conclude that under RPE, the manager’s expected payment is increasing in $\theta_w$. As a consequence, we have that as $\theta_m \geq \theta_w$, $\frac{\partial \Psi^2}{\partial \theta_w} \geq 0$. That is, the manager’s expected payment is hump-shaped in $\theta_w$ with a maximum at $\theta_w = \theta_m$. From this analysis, it is clear that the manager with $\theta_m = \theta_h$ prefers $\theta_w = \theta_l$, whereas, the manager with $\theta_m = \theta_l$ prefers $\theta_w = \theta_h$.■

Proof of Proposition 5.

From (17) and (18), we have

$$V(e = 1) - V(e = 0) = \theta_m \Delta R - \Psi^2 + u^{-1}(\bar{u}).$$

Whether effort is induced or not depends on $\Psi^2$ is smaller or greater than $\theta_m \Delta R + u^{-1}(\bar{u})$. Since $\theta_m = \theta_w$, the optimal contract follows IPE. Thus, $\Psi^2 = \Psi^1 = \theta_m (c/\theta_w + \bar{u}) + (1 - \theta_m)\bar{u} = (\theta_m/\theta_w) c + \bar{u}$. Under the assumption (14), no effort is induced when $\theta_m = \theta_w$. Recall that $\Psi^2$ has a maximum at $\theta_w = \theta_m$ and is hump-shaped in $\theta_w$. There must exist $\theta^*_w$ and $\bar{\theta}_w$ such that $\Psi^2(\theta^*_w) = \Psi^2(\bar{\theta}_w) = \theta_m \Delta R + u^{-1}(\bar{u})$. ■
Proof of Proposition 6.

Given any effort \( e^* \), we minimize the expected payment subject to (19) and (20). The Lagrangian of the problem is

\[
\mathcal{L} = (\theta_m e^*)^2 v^{SS} + \theta_m e^*(1 - \theta_m e^*) \left( v^{SF} + v^{FS} \right) + (1 - \theta_m e^*)^2 v^{FF}
\]

\[
- \lambda_1 \left[ \theta_w e^* u(v^{SS}) + (1 - \theta_w e^*) u(v^{SF}) - \overline{w} - C(e^*) + C'(e^*) e^* - \frac{C'(e^*)}{\theta_w} \right]
\]

\[
- \lambda_2 \left[ \theta_w e^* u(v^{FS}) + (1 - \theta_w e^*) u(v^{FF}) - \overline{w} - C(e^*) + C'(e^*) e^* \right].
\]

The nature of the problem is no different from the binary effort case. Setting the slope of the isocost line equal that of each constraint, we obtain:

\[
\frac{1 - \theta_m e^*}{\theta_m e^*} = \frac{(1 - \theta_w e^*) u'(v^{SF})}{u'(v^{SS})} = \frac{(1 - \theta_w e^*) u'(v^{FF})}{u'(v^{FS})}.
\]

The manager’s choice of JPE or RPE is, given any effort \( e^* \), determined by \( \theta_m \geq \theta_w \). Also, the above equations can be rewritten as

\[
\frac{u'(v^{SF})}{u'(v^{SS})} = \frac{u'(v^{FF})}{u'(v^{FS})} = \frac{\theta_w (1 - \theta_m e^*)}{\theta_m (1 - \theta_w e^*)}.
\]

From this, it is immediate that \( v^{SS}/v^{SF} \) and \( v^{FS}/v^{FF} \) are decreasing with \( e^* \) with \( \theta_m > \theta_w \), while \( v^{SS}/v^{SF} \) and \( v^{FS}/v^{FF} \) are increasing with \( e^* \) with \( \theta_m < \theta_w \).

Proof of Proposition 7.

The minimum payment contract to worker \( i \) is given by

\[
\min_{v_i^1} \left[ \theta_m v_i^{SS} + \theta_m (1 - \theta_m) v_i^{SF} + \theta_m (1 - \theta_m) v_i^{FS} + (1 - \theta_m)^2 v_i^{FF} \right]
\]

subject to

\[
\theta i_1 \theta i_1 u(v_i^{SS}) - u(v_i^{FS}) + \theta i_1 (1 - \theta i_1) u(v_i^{SF}) - u(v_i^{FF}) \geq c \quad (35)
\]

\[
\theta i_1 \theta i_1 u(v_i^{SS}) + \theta i_1 (1 - \theta i_1) u(v_i^{SF}) + (1 - \theta i_1) \theta i_1 u(v_i^{FS}) + (1 - \theta i_1) (1 - \theta i_1) u(v_i^{FF}) - c \geq \overline{w} \quad (36)
\]

When (35) and (36) are solved together, the two constraints are reduced to (21) and (22). Following our previous analysis, we get

\[
\frac{1 - \theta_m}{\theta_m} = \frac{(1 - \theta i_1) u'(v_i^{SF})}{u'(v_i^{SS})} = \frac{(1 - \theta i_1) u'(v_i^{FF})}{u'(v_i^{FS})}.
\]

The results in Proposition 7 follow immediately.
References


