

**Option Investor Rationality Revisited:
The Role of Exercise Boundary Violations**

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Abstract

Do option investors rationally exercise their options? Numerous studies report evidence of irrational behavior. In this paper, we pay careful attention to intraday option quotes and reach the opposite conclusion. An exercise boundary violation (EBV) occurs when the best bid price for an American option is below the option's intrinsic value. Far from being unusual, we show that EBVs occur very frequently. Under these conditions, the rational response of an investor liquidating an option is to exercise the option rather than sell it. Empirically, we find that the likelihood of early exercise is strongly influenced by the existence and duration of EBVs. Not only do these results reverse standard theory on American option valuation and optimal exercise strategy, but they also suggest that the ability to avoid selling at an EBV price creates an additional source of value for American options that is unrelated, and in addition to, dividend payments. This additional value may help explain why American options appear overpriced relative to European options.

Keywords: American options; option exercise; early exercise boundary; investor rationality

JEL Classifications: G13, G12, G14

NOTE: An earlier version of this paper was titled: "Exercise Boundary Violations in American-Style Options: The Rule, not the Exception."

The rationality of option investors has been the subject of considerable research, and the general consensus is that their behavior departs significantly from rationality. For example, Diz and Finucane (1993), Poteshman and Serbin (2003), Pool, Stoll, and Whaley (2008), Hao, Kalay, Mayhew (2010), and Barraclough and Whaley (2012) all conclude that a significant fraction of options investors use irrational exercise strategies for American options. In this paper, we revisit the question of option investor rationality and find that when market frictions are properly considered, the behavior of option investors is both consistent with rationality and inconsistent with traditional options theory.

Following Merton (1973), the conventional wisdom is that early exercise of a call on a non-dividend paying stock is irrational because it yields only the call's intrinsic value ($S - K$) and thus forfeits its time value. This perspective, however, does not consider trading frictions. We document that among options that are in-the-money and might be exercised, the option's bid price is frequently *lower* than the option's intrinsic value. We call this counter-intuitive situation an Exercise Boundary Violation (EBV).

EBVs play a critical role in assessing investor rationality. The simple choice for an investor seeking to liquidate an option holding is either to sell or exercise their position. While theory holds that early exercise is irrational, under EBV conditions early exercise is the rational choice since the option holder receives a greater payout than from selling the option. Selling under EBV conditions gives the counterparty an arbitrage profit from immediate exercise.

Here is a typical EBV. At 1:57 P.M. on April 24, 2017, Exxon-Mobil stock was quoted at \$81.16 bid and \$81.17 ask. The May and June 75 strike calls would expire in 25 and 53 days, respectively. Both options were in the money by \$6.16, but the highest bids across all the option trading venues were only 5.85 and 5.90, respectively. The "irrational" decision to liquidate a

long position in either call by exercise would have yielded about 5% more than selling it in the market. This example is not an unusual case. April 24, 2017 was a typical day and XOM options are among the 1% most actively traded single stock options.

In this paper, we examine a large cross-section of U.S. equity options using price and quote data observed at one-minute intervals. We find that EBVs are frequent, economically large, and very hard to avoid. Overall, 48.6% of all in-the-money call option bid quotes were below intrinsic value. These EBVs are highly persistent and averaged \$0.31, large enough that even retail investors could save a significant amount by exercising their options instead of selling them. A similar pattern appears in put options where the EBVs remain frequent, persistent, and economically large.

Evidence of boundary violations has been noted in option market studies dating back at least to Klemkosky and Resnick (1979) and further developed by Diz and Finucane (1993), Finucane (1997), and Engstrom (2002). Despite this history, our understanding of EBVs is very limited. It is well known that option trading costs are large, but many of our colleagues are surprised by the frequency and magnitude of EBVs. In addition, this literature is limited by its reliance on closing prices. For example, a wider bid-ask spread at the close of trading could generate an EBV even if there were many minutes without EBVs during the regular trading day.

This paper provides the first systematic examination of exercise boundary violations and their link with investor behavior. Our results provide clear support for the rationality of option exercise. We find that the existence and duration of boundary violations have strong predictive power for the cross-section of individual option exercises. In addition, remarkably few option trades occur at sub-optimal prices. Across all in-the-money option trades in our sample, only

about 2% of them occur at prices that are below the exercise boundary value. Both findings suggest that option investors rationally incorporate EBVs into their trading and exercise decisions—even though their actions are inconsistent with option pricing theory. Even so, the dollar loss from the sub-optimal trades appears substantial. An upper bound estimate is \$39 million for the month of March 2010. By comparison, this is about three times the average monthly loss reported by Barraclough and Whaley (2012) from failure to exercise put options optimally.

An important implication of our findings is in the area of option pricing. In theory, the only value from early call exercise comes from the ability to exercise just before a stock goes ex-dividend. If an investor must exit an option position before maturity when EBVs are pervasive, the holder of an in-the-money European option expects to lose both the remaining time value and the EBV, while a rational American option holder could exercise, giving up the time value but avoiding the EBV. Thus, an American option is worth more than a European call with the same terms, independent of dividend payout. A related paper (Figlewski 2017) shows that the difference can easily be more than the value of rational American exercise of a deep in the money put, or of a call on a stock about to go ex-dividend. Empirical studies by Valkanov, Yadav, and Zhang (2011) and others find that the observed difference between American and European option prices is larger than can be explained by early exercise models. Our findings offer a potential resolution to this anomaly.

This paper is most closely related to Jensen and Pedersen (2016) who also find evidence of rational early exercise. The papers differ in three areas. First, our approach is extremely simple and relies only on the relation between the strike price and the option bid price. In contrast, Jensen and Pedersen develop a model that predicts when early exercise is optimal and then solve

their model using numerical methods. All tests of rationality involve a joint test of investor behavior and the underlying pricing model. While their approach is very interesting, the accuracy of their pricing model is uncertain. For example, EBVs are common in regions of their parameter space where their model predicts no early exercise and, in their early exercise regressions, EBVs remain the most statistically important factor even after controlling for the parameters of their pricing model.

Second, the model in Jensen and Pedersen (2016) is limited to call options and does not apply to put options. In contrast, our approach extends easily to put options. In our analysis, we control for the potential early exercise of American puts but nevertheless find the same results as with call options. The existence and duration of EBVs remain strong predictors of put option early exercise.

Finally, our intraday data is better suited for testing hypotheses about investor rationality. Like most other studies, Jensen and Pedersen (2016) use closing prices and quotes. Battalio and Schultz (2006), however, show that reported closing option prices are often anomalous and may even show arbitrage violations due to non-synchronous reporting of stock and option quotes. With synchronous intraday data we are able to measure boundary violations much more precisely and quantify how they behave during the trading day. This distinction is important because we find the frequency of EBVs within the trading day is a strong predictor of early exercise.

The rest of the paper is structured as follows. Section 2 provides a review of the literature. Our data is described in Section 3. Section 4 presents the data on exercise boundary violations for calls and puts. Since EVBs are critical to our argument supporting investor rationality, we go

into considerable detail showing that EBVs are pervasive, economically large and highly persistent throughout the trading day.

Section 5 looks at option trades. We find that relatively few trades occur at EBV prices and that the average EBV associated with these trades is comparatively small. Section 6 looks at the option holder's alternative liquidation strategy, exercise. We find that non-dividend related early exercise is relatively common among both customers and market makers, although the frequency of early exercise is greater for the market makers. We then estimate logit models to examine factors that determine the likelihood of early exercise. Exercise by both customers and market makers is highly dependent on the size and duration of EBVs, along with option moneyness. Exercise of puts is also strongly influenced by whether the stock price is above or below the put option theoretical early exercise boundary.

In Section 7 we discuss several ways in which EBVs may affect options theory and practice, including how American options should be priced in theory and how option positions should be marked-to-market in practice, as well as correct calculation of implied volatilities and use of the bid-ask spread as a measure of market liquidity. Section 8 concludes.

An important issue that we do not attempt to address in the current paper is why the best bid prices in the market are so frequently below fair value, to the point that they entail an arbitrage profit for the buyer. Competition among dealers and other liquidity providers ought to force the bids up, but it does not. Yet, discussions with options market makers did not uncover any hidden transactions costs or major impediments to option exercise. We also do not derive an alternative American option pricing model to take proper account of the liquidity value of being able to exit

an option position early without giving up an EBV. Such a model is developed in Figlewski (2017).

2. Boundary Violations and Option Exercise in the Literature

Tests of investor rationality are typically joint tests of both rationality and an underlying asset pricing model. As noted in Diz and Finucane (1993), option exercise decisions provide an attractive area to test rationality because investor choice is reduced to selecting the best alternative between exercising and selling the option. Most studies of exercise rationality conclude that investors frequently make irrational decisions. For example, Pool, Stoll, and Whaley (2008) and Hao, Kalay, and Mayhew (2010) examine the relation between dividends and exercise behavior. In theory, an in-the-money American call should be exercised just before the ex-dividend date if the dividend payment is sufficiently large, but these studies find that about 50% of the calls for which this would be optimal are not exercised. Barraclough and Whaley (2012) provide a comprehensive analysis of option exercise. They find, as we do, that call options are frequently exercised early for reasons unrelated to dividends, yet some deep in-the-money put options fail to be exercised early. Diz and Finucane (1993) and Poteshman and Serbin (2003) document a tendency for option investors to exercise call options when they would have received more by selling the call.

Despite this research, very little is known about EBVs and their link to early exercise decisions. Early option market studies such as Bhattacharya (1983) noted the occurrence of exercise boundary violations and Bates (1995) summarizes this literature. None of these studies systematically examine EBVs. Subsequent work by Diz and Finucane (1993), Finucane (1997)

and Engstrom (2002) note a link between EBVs and exercises, but do not analyze the relationship in a structured manner.

An important limitation of these studies is that their data is from the 1980s or earlier. This predates significant structural changes in the options and equity markets and it is not obvious their EBV findings apply to today's markets. For example, Angel, Harris, and Spatt (2011) document that equity commissions and bid-ask spreads costs have fallen dramatically while trading volume and market depth have strongly improved. Similarly, studies by Mayhew (2002), de Fontnouvelle, Fische and Harris (2003), and by Saraoglu, Louton, and Holowczak (2014) document a decline in option bid-ask spreads from changes in listing eligibility and reductions in the minimum required tick size. Even when restricted to closing prices, Christoffersen, Goyenko, Jacobs, and Karoui (2015) report large improvements in option liquidity over the 1996-2007 period.

The impact of these changes can be illustrated by contrasting our sample with Finucane (1997). His sample spans the calendar years 1988 and 1989 and contains 9.1m call option contracts exercised. In contrast, we observe 13.7m call option exercises (including dividend-related exercises) in just the first 20 calendar days of March 2010. From the late 1980s to 2010, the monthly number of contracts exercised has risen more than 100-fold. Similarly, after excluding early exercises associated with dividends, Finucane reports that 38.4% of contracts were exercised when the bid price exceeded the strike price. The comparable frequency for these apparently irrational exercises falls to 1.8% in our sample.

Recent research is more supportive of the idea that market frictions can make it rational to deviate from conventional early exercise rules. In a theoretical model, Duffie, Liu, and

Poteshman (2005) formalize the intuition that an American option should be exercised if the price falls below intrinsic value. But they note that an investor's welfare may be higher by retaining the option if markets are incomplete and trading costs are sufficiently large. While a large EBV is not a sufficient reason to exit an option position, it is a sufficient reason to favor exercise over selling the option.

Costly hedging plays a key role in the models of Figlewski (1989) and especially Jensen and Pedersen (2016). Figlewski simulates an options market with rebalancing costs and shows that a bid-ask spread sufficient to compensate the market maker for the risk in hedging his position can lead to bid prices below intrinsic value. Jensen and Pedersen consider alternative strategies for liquidating an American option position, either by exercise or by synthetically eliminating its risk exposure by delta hedging through expiration. This is the theoretically correct way to avoid the problem of an illiquid or nonexistent options market. They develop and test an elegant continuous-time model in which the costs associated with short sales and funding long positions can drive prices below intrinsic value. Their empirical results indicate that these costs are large enough in practice to make early exercise often the optimal liquidation strategy, and that their model can explain some of the observed "non-rational" early exercise.

But Jensen and Pedersen (2016), Finucane (1997), and most other researchers analyze only closing prices and quotes. The results are informative, of course, but as Dennis and Mayhew (2009) found, tests of option pricing models are sensitive to microstructure issues, and Battalio and Schultz (2006) and others have shown that wider spreads and anomalous quotes are relatively common in end-of-day prices. Moreover, closing prices do not reveal what trading possibilities may have been available earlier in the day. Use of intraday data allows us to show

that holders of both calls and puts face large and persistent exercise boundary violations throughout the trading day.

The underlying theme in this line of research is that traders often behave irrationally with respect to their exercise decisions. Our position is the opposite: we argue that once EBVs and other market frictions are taken into account, much of the behavior that appears irrational in theory can be seen to be rational in practice. Jensen and Pedersen (2016) report findings consistent with our interpretation, but for a different reason. In their model, early exercise becomes optimal if market frictions are sufficiently large. In our approach, early exercise is rational if the bid price is below the intrinsic value. A task for future work is to examine whether their model could explain the EBV patterns we observe in the data. This may be challenging because near term, at-the-money (but still in-the-money) call options appear to be below the price/strike price barrier that would trigger early exercise, yet over one-third of these option quotes have EBVs.

If an option holder can always realize its theoretical value by selling (or delta hedging) in the market, an American call is only better than a European call because it allows exercise just before a stock goes ex-dividend. But when Valkanov, Yadav, and Zhang (2011) and Dueker and Miller (2003) tested this theoretical principle in option market prices, they found early exercise premiums of 3-6% for call options and 4-10% for put options, much too large to be explained by dividend-related early exercise alone.¹ Option traders apparently believe that American exercise has some additional value unrelated to dividends.

¹ See also Jorion and Stoughton (1989), McMurray and Yadav (2000), Sung, (1995), Unni and Yadav (1998), and Zivney (1991).

3. Data

Our sample covers options on 2945 stocks across all expiration months for 21 trading days during March 2010. Data from March 22 are corrupt and excluded from the analysis. We limit the sample to options that are in-the-money at the start of the trading day and all ETFs are excluded. In order to focus exclusively on "irrational" exercise, we eliminate all observations for dividend paying stocks on the day preceding the ex-dividend day. We also exclude all observations on the day of expiration. Each stock will typically have both call and put options with multiple exercise prices, which yields an average of 124,652 individual options for each trading day.

The data come from four sources. Option trades and quotes are from Livevol. Trade data include the date, time, and size of each transaction. For option quotes, Livevol provides the national best bid and offer prices across the eight option exchanges, reported at one-minute intervals. There are 390 trading minutes per day for each option, which yields a data matrix of roughly 1.02 billion elements: 124,652 (options) x 21 (days) x 390 (minutes). The specific contracts in the sample change over time as option moneyness varies with stock price changes, and expiration dates pass, and not every option series is quoted every day. In addition, we eliminate very low liquidity contracts with fewer than 180 daily quotes and all observations with a bid price of zero. This reduces our data matrix to approximately 671 million elements.

Intraday tick data for equity trades and quotes to match the options sample come from the NYSE TAQ database. We use CRSP to identify and exclude ex-dividend days. Finally, data on daily

open interest and exercises from both customer and market maker accounts come from the Option Clearing Corporation.²

To reduce the dimensionality of the option quotes, we classify the strike price / stock price ratio into three moneyness categories:

"Deep ITM" ($X/S < 0.7$ for calls and $1.3 < X/S$ for puts);

"Mid ITM" ($0.7 < X/S < 0.9$ for calls and $1.1 < X/S < 1.3$ for puts); and

"Near ITM" ($0.9 < X/S < 1.0$ for calls and $1.0 < X/S < 1.1$ for puts).

Similarly, we set up three maturity categories: "near term" (1 month or less); "medium term" (2-4 months); and "long term" (over 4 months). Within this $3 \times 3 \times 2$ partition, the number of observations per cell ranges from 12.1 million to 69.9 million.

Note that for an option falling into one of these categories, the likelihood that the option will remain in-the-money at expiration depends on option maturity and the volatility of the underlying stock. For example, a one-month call on a stock with $X / S = 0.9$ and annual volatility of 40% is in-the-money by less than one standard deviation. If $X / S = 0.8$, the midpoint of the Mid ITM bracket, a three-month call on that stock is still only 1.1 standard deviations in-the-money. Prices for these options should embed substantial optionality, but as we show below, they still have a strong chance of being subject to EBVs.

Historically, options priced below \$3.00 traded on tick size of \$0.05 and options priced above \$3.00 had a tick size of \$0.10. In 2007, option exchanges initiated the penny pilot program in

² The OCC data were generously provided to us by Robert Whaley of Vanderbilt University.

which they reduced the tick size for more liquid options.³ In our sample, 178 option classes were included in the penny pilot program. For these options the tick size was reduced to \$0.01 for prices below \$3.00, and \$0.05 for options above \$3.00.

4. Frequency, magnitude and persistence of boundary violations

In this section we explore how common EBVs are as a function of moneyness and maturity, how large they are economically, and how persistent they are in terms of how long an option holder facing an EBV bid at the beginning of the day would have to wait to see a bid at or above intrinsic value.

4.1 Frequency and Size of Boundary Violations

Table 1 presents summary statistics for exercise boundary violations in our sample. Panel (a) shows results for call options and Panel (b) presents puts. The largest numbers of quotes are for the mid moneyness and long term maturity bins, largely due to the width of the categories, but both panels in Table 1 show high quote frequency for all categories, with the smallest cell containing more than 20 million observations for calls and 12 million for puts.

For near term Deep ITM calls, 98.73% of all bids were below intrinsic value and 99% of all options in this category had EBV bids at least some of the time. For near term Mid ITM contracts, still 92.77% of all bid quotes were EBVs and more than 98% of all contracts had them. Even for near maturity calls that were only a little in-the-money, the best bid in the market was below intrinsic value more than a third of the time. Note that the call in the XOM option

³ Saraoglu, Louton, and Holowczak (2014) examine the impact of the penny pilot program on options trading costs.

example above was in the Near ITM moneyness bucket, less than 8% in the money, with $X/S = 0.92$. The XOM June75 contract was also in the middle maturity bucket.

The frequency of boundary violations falls with longer maturity and lower moneyness as one would expect. Both factors increase an in the money American option's time value, so even if a bid is well below a long-dated option's theoretical fair value it is less likely to be below intrinsic value and cause an EBV. Even so, for Deep ITM long maturity calls, about 60% of market bid quotes are EBVs. Interestingly, EBVs are somewhat less common for puts, particularly as maturities lengthen, even though in theory puts may be rationally exercised early because their theoretical time value can fall to zero.

Table 1 also provides the mean boundary violation in each maturity and moneyness bucket. We report the unconditional EBV, which is the average of EBVs across all observations, including a value of zero for all non-EBV observations. We also report the conditional EBV, which is the average EBV across all quotes that are EBVs. For comparison, Table 1 also contains the average option bid-ask spread.

A useful comparison is to contrast the EBVs with the quoted bid-ask spread. While there is variation across the buckets, the EBVs are a material fraction of the spread. For example, among the near-term deep in-the-money calls, the average bid-ask spread is \$0.72 and the average conditional EBV is \$0.35, so the boundary violation represents nearly half of the spread. A similar pattern occurs in the near-the-money options where the average spread is \$0.33 and the conditional EBV is \$0.14.

A separate analysis of options that are partitioned into those with prices above \$3.00 and below \$3.00, and into pilot study options and non-pilot study options is presented in a supplementary

Appendix that is available from the authors. It reveals significant differences in liquidity across the subgroups, but they generally show the same pattern of EBVs as in Table 1.

Two other considerations about Table 1 should be noted. First, our results are not specific to this time period. In untabulated analyses, we find very similar results in August 2008. Second, the EBVs cannot be easily explained by short sale costs. Asquith, Pathak, and Ritter, (2005), and Kolasinski, Reed, and Ringgenberg (2013) argue that short sale costs are not material for the vast majority of stocks. However, if only a small fraction of stocks have material short sale costs, they cannot explain why almost all near-term, Deep ITM options exhibit EBVs.

The distribution of EBVs for different moneyness and maturity buckets is of considerable interest. Figure 1 plots the percentage of quote minutes that exhibit a boundary violation of a given dollar amount for the near term options. Figure 2 plots the same information for the medium maturity contracts. Note that the horizontal axis starts with an EBV of \$0.05. For example, for near term calls in the middle moneyness bucket, Figure 1 shows EBV was at least \$0.05 about 85% of the time, and over \$0.25 about 30% of the time. EBVs for the comparable puts were similar but a little lower. In Figure 2, the middle moneyness calls with 2 to 4 months to maturity showed EBVs of at least \$0.05 a little less than half the time, and more than \$0.25 about 20% of the time. Large EBVs for longer term puts were less common than for calls.

4.2 Persistence in Exercise Boundary Violations

Boundary violations need not prevent an option holder from selling at a price above intrinsic value as long as there are periods during the day without an EBV. This cannot be established by looking only at closing prices. To assess EBV persistence, we select 10:00 a.m. as the initial observation time, because we can be confident that the options market is fully open by that time

and all strikes and maturities can be traded freely.⁴ If the 10:00 a.m. quote is a boundary violation, we go forward in time and search for the first occurrence of a non-EBV bid, stopping the search process at 4:00 p.m. With this starting time, the maximum duration of a boundary violation is 360 minutes.

Table 2 uses the same partitions for option maturities and moneyness as Table 1. The results show a strong persistence in boundary violations. For near term Deep and Mid ITM calls and puts, more than 90% had EBV bids at 10:00 a.m., and these persisted for many hours on average. For example, the average EBV persistence for the near-term Mid ITM contracts is 297 minutes for calls and 296 minutes for puts. A trader who observes an EBV at 10:00 would have to wait, on average, until almost 3:00 PM for the EBV to disappear. A similar pattern is even evident for the long-term options. EBVs are less common in options with more than 4 months to maturity, but when they occur, their magnitude and duration are similar. For example, the violation rate is 59.51% for the Deep ITM calls, but under 1% for the Near ITM contracts. Nevertheless, the persistence of the violations remains high, 266 and 178 minutes, respectively.

We will discuss transactions costs and other impediments that affect option trading in the next section.

5. Boundary Violations and Option Trades

5.1 Commissions, margins and settlement issues

The optimal exercise strategy for an investor to liquidate an American option position before expiration depends on the actual transactions costs and the other impediments to trading that he

⁴ Similar results were found when the observations were sampled at 11:00, 12:00, and 2:00.

faces. Securities firms catering to retail traders charge a wide range of commissions and fees. We discuss how two of the largest handle early exercises.

Interactive Brokers (IB), one of the lowest cost firms, provides full electronic information and execution facilities online. There is no charge for exercising an option, which can be done at any time simply by clicking on a computer screen. When the "exercise" button is clicked, IB immediately places the stock in the investor's account and it can be sold in the market. Within a few seconds the option holder is able to see the market stock price, exercise his call options, and sell the stock to lock in the current intrinsic value. IB's commission for a stock trade is \$0.005 per share (\$1 minimum). Thus, an investor can exercise his American calls at any time and receive the intrinsic value at the current stock price for a total cost of 50 cents per contract (the \$1 minimum corresponds to 200 shares, i.e., two option contracts). Exercise of a put that requires first buying the stock to be delivered is similar, and it is even cheaper if the put is protecting stock that is already owned, since no stock trade is needed. Commissions at Charles Schwab, another large low-cost retail broker are somewhat higher, but still low enough that they are easily outweighed by even a very small EBV.

The alternative strategy of holding call options unexercised, but getting rid of their market exposure by delta-hedging with the stock through expiration day is more costly than early exercise. The commission to trade the stock in setting up the hedge will be about the same as to sell out the stock acquired by exercising a call, but depending on the path of stock prices, maintaining delta neutrality can entail actively rebalancing the hedge over time and many more stock trades. In addition, maintaining a short position in the stock requires paying the stock borrow fee and posting collateral, initially equal to 50% of the stock value, in a margin account for the remaining life of the option. This strategy may be appropriate for the holder of a

European call who otherwise would need to sell at the EBV bid, but it will normally be dominated by exercise for an American option.⁵

5.2 Boundary Violations and Option Trades

An investor who sells an option at a price below the exercise boundary is (perhaps unknowingly) leaving money on the table. A natural question to ask is how often does that occur? Although the displayed quotes exhibit sustained boundary violations, investors might either exercise their options instead of selling or trade strategically by picking the right times to avoid, or at least minimize, the boundary violations. To see whether option traders avoid the inferior prices associated with the worst boundary violations, we examine all option trades in our sample that occur above, and below, the boundary violation price, using the bid and ask quotes in effect at the exact time of the trade.⁶ For trades below the EBV price, we compute the dollar amount of the deviation from the EBV price.

Table 3 provides summary statistics for option trades in our sample. Option exercises, by definition, are excluded. Note that the relative weighting in this table is by the number of transactions, unlike Tables 1 and 2 in which each option has equal weight. If one option series

⁵ Delta hedging, in theory, allows recovery of an option's full value, not just intrinsic value, which makes it better than exercise in a frictionless world. The intrinsic value is embedded in the difference between the current stock price and the strike, but replication of the optionality value comes from (frictionless) continuous rebalancing of the hedge. Frequent rebalancing in the real world quickly leads to large transactions costs, and it does not fully eliminate risk. In most cases, exercise of an American option will be cheaper, easier, and less risky than attempting to exit the position synthetically by hedging away its market exposure.

⁶ It might seem reasonable to measure execution quality by the difference between the trade price and the quote midpoint and then examine whether execution quality improves when the boundary violations are large, because it becomes more possible for investors to trade inside the spread. Unfortunately, this idea does not work because investors trying to escape EBVs can be expected to try placing limit orders inside the marketmakers' spread, which alters the recorded best prices. We argue below that in an EBV it may be more appropriate to treat the option's intrinsic value as the market's best bid.

has 10 trades in a day and the second has only one, the first option will generate 10 transaction observations, while the second option generates just one.

Panel (a) shows EBV trading of calls, and Panel (b) of puts, using the same maturity and moneyness breakdown as before. Panel (c) aggregates across categories to report the total deviation from selling at EBV prices, which totals about \$39 million for the month and represents an upper bound on the opportunity costs associated with the failure to exercise. Most of this was due to call option trades, about \$37.2 million versus \$1.8 million from puts.

In Panel (a) dollar EBV deviations from trading calls occur in each sub-group, and range from three to nine cents per share. The largest dollar apparent losses, however, are concentrated in the near term options, as one expects, because EBVs are larger and more prevalent. Among the near-term, Mid ITM calls, 12.13% of trades and 28.06% of volume occurred below the boundary price. This corresponds to a potential opportunity loss averaging \$515.25 per trade.

Selling at an EBV price can occur from an investor either closing out an existing long position, or from establishing a new short position. Our data cannot distinguish between these two motives, but in either case, it is difficult to justify selling an option at a price that gives the counterparty the ability to exercise it immediately for a profit.⁷ Our estimates of the dollar loss from EBV trades should be viewed as an upper bound.

Yet even among options that exhibit EBVs more than 90% of the time, only 12% or so of the trades are done at EBV prices. Interestingly, a higher proportion of trading volume than of trades is done at EBVs, which means those trades are larger than average. It might seem that a

⁷ One possible reason for establishing a short position at an EBV price is to circumvent short sell rules. The SEC is critical of these strategies. See SEC Administrative Proceeding 3-14847.

possible reason for this is that call exercise followed by selling the shares in the market depends on adequate liquidity in the underlying stock. The average EBV trade size for near term Mid ITM options is 130 contracts, which corresponds to 13,000 shares, while the average EBV is only \$0.04. Depending on the stock, it could well be that dumping 13,000 shares on the market at once could knock the price down at least 4 cents.

This reasoning could justify selling an option to the market maker at an EBV price, but it is not really a good explanation for exercising only at maturity, since the stock will have to be sold in the market whenever an in the money call is exercised. If limited liquidity in the stock makes it rational to sell at an EBV before expiration, the option holder should also find it better to sell to the market maker than to exercise on expiration day. The market impact to liquidate the stock is not avoided by waiting until option maturity.

The long maturity Mid ITM calls, which averaged nearly 1500 contracts per trade but very few exercises, as we will see in the next section, would certainly be consistent with this explanation.

In contrast, put options exhibit a much smaller fraction of EBV trades and volume. Although the boundary violations for several of the put sub-samples are around \$0.40 per share, only a very few such trades were done, which makes the overall loss much smaller.

The summary of these results in Panel (c) shows that investors generally avoided selling at sub-optimal prices. Of all in-the-money option trades, only 11.52% of trading volume and 2.01% of trades occurred below the boundary price. On average, the transaction price was about \$0.04 below the exercise boundary price, corresponding to a loss of about \$521.81 per trade. The evidence suggests that investors adjust their trading behavior to the boundary violations, although it is difficult to prove unambiguously. Comparing the average EBV for trades against

the average EBV quotes for the same option categories in Table 1 reveals that investors who did trade managed to avoid the worst of the boundary violations, consistent with getting some price improvement on their trades. Given how pervasive boundary violations were in the quotes, surely more than 2% of trades and 11.5% of volume would be below the boundary price if traders were not paying attention to EBVs. At a minimum, these results are consistent with investors responding rationally to EBVs.

Muravyev and Pearson (2014) suggest an alternative explanation for the lower EBVs associated with option trades. They show that the underlying stock price can exhibit considerable variation without triggering a change in the option quotes and investors incorporate this information to strategically time their trades to lower their effective trading costs. This is consistent with observing a smaller EBV from actual trades than from option quotes.

Nevertheless, the aggregate deviation from EBV prices of \$39 million for just one month is a material amount and at least some of it could have been saved by exercising the options instead of selling them. By comparison, the opportunity loss from these trades exceeds the losses documented in Pool, Stoll, and Whaley (2008) from the failure to exercise call options before ex-dividend days (about \$4 million per month), and the losses reported in Barraclough and Whaley (2012) from the failure to exercise put options (about \$13 million per month).

6. Boundary Violations and Option Exercises

Section 4 established that exercise boundary violations are frequent, persistent, and economically significant. In this section we examine how boundary violations affect exercise decisions.

Previous studies reported evidence of early exercise among exchange-traded American-style options but without focusing on EBVs.⁸ Given that there appears to be little awareness of the significance of boundary violations in the literature and the established theory emphasizes that early exercise is sub-optimal, it is an open question whether these exercises are mostly mistakes or rational responses to pervasive EBVs.

To examine this question, we analyze the option exercises during our sample period. The available data report only the total number of contracts exercised on a given day, so 20 contracts could be one exercise for all 20, or 20 exercises for 1 contract each. We therefore define the occurrence of one or more exercises for a given contract on a particular day as an Exercise Event.

Customers and market makers have different reasons for holding or exercising in-the-money option positions. A customer with Deep ITM calls most likely bought them for lower prices at some earlier time and may now want to exit the position and take her profit. A market maker rarely has a long term investment motive and will typically acquire an ITM option position by taking the opposite side of a customer's trade. If he buys calls or puts at an EBV price, he may well simply exercise immediately and take the arbitrage profit, rather than holding illiquid high priced options in inventory. Alternatively, he may retain the options if there is a reasonable chance to sell them at his ask price before too long, especially if the delta and other Greek letter exposures are useful in hedging the rest of his trading book. A customer with deep in-the-money puts may well be holding them to protect the value of a long position in the underlying stock.

⁸ Examples include Diz and Finucane (1993), Overdahl and Martin (1994), Finucane (1997), Engstrom (2002), Pool, Stoll, and Whaley (2008), Hao, Kalay, and Mayhew (2010), Barraclough and Whaley (2012), and Jensen and Pedersen (2016).

Since liquidating the puts would ruin the hedge, she may be more willing to hold onto them through expiration, even when theory might suggest early exercise would be optimal.

Summary statistics for exercise events are presented in Table 4. None of these exercises should be dividend related, since we exclude all observations from the day before an ex-dividend day.

We also exclude the final day before expiration in order to look only at early exercise.

Panel (a) shows that early exercise of calls unconnected to dividend payout is frequent. The Total column at the far right shows that in March 2010 there were 10,334 exercise events involving 950,683 contracts. As expected, exercise is related to option maturity and moneyness. Near term calls in the Deep and Mid ITM categories experience the most exercises, by both customers and market makers. Customers have more exercise event days but exercise fewer contracts than market makers overall.⁹

Panels (b)-(d) of Table 4 provide the exercise statistics for put options. Unlike calls, if an American put in a frictionless market goes deep enough in-the-money, it becomes rational in theory to exercise it early. For any given put, on every day t there is an early exercise stock price S_t^* such that if the stock price is below S_t^* , the put is worth less than its intrinsic value and it should be exercised. For each put observation in our data, we compute the critical stock price for (theoretically) rational early exercise in a Binomial model. The put is then classified as one that should, or should not, be exercised early. Note that exercise when the stock is below S_t^* does not necessarily mean the investor followed the theoretically optimal strategy. That would require exercising the put the first time the boundary is breached. Also, it is important to recognize that

⁹ Barraclough and Whaley (2012) also found that market makers exhibit early exercise behavior that runs counter to conventional theory.

there is no direct connection between the critical stock price and an EBV. A put may be in the range where its fair value is above intrinsic value and theoretically it should not be exercised, yet if the best available bid in the market is an EBV, the option holder who must liquidate still does better by exercising the put than by selling it.

Panel (b) shows that there were fewer early exercises of put options than call options, 4324 versus 10,344 exercise events, although they exhibit similar patterns across maturity and moneyness as in Panel (a). One reason for the difference in exercises, as shown in Table 1, is that for the near-term Deep and Mid ITM contracts, there were simply many more quotes for calls than for puts.¹⁰ The ratio of market maker exercises to total exercises is also higher for puts (78.8%) than for calls (58.8%). This would be consistent with the idea that customers may be relatively more reluctant to exercise puts that are hedging a stock position. There is little exercise of either long maturity calls or puts.

Comparing Panels (c) and (d), put option exercises are divided evenly between those that should have been exercised early in theory and those that should not. Of the 267,604 total put exercises, just 51.7% occurred when the stock price was below the early exercise price barrier. Panel (c) shows that, relative to customers, market makers were more likely to exercise when it was theoretically correct to do so.

¹⁰ Exchanges introduce option strikes around the current underlying stock price, and as the stock price moves, new strikes are introduced while existing options may go deep in or out of the money. A moderate advance in the stock market in the months before March 2010 left many of the previously introduced calls deep in-the-money and the corresponding puts out-of-the-money (and out of our sample).

6.1 Estimating the Likelihood of Early Exercise

The previous section demonstrated that despite being irrational in theory, early exercise is common in the real world. This section investigates early exercise in a more structured framework by fitting logit models for the probability of option exercise. The sample consists of all individual options that have either positive trading volume or a change in open interest on a given day. We exclude any options with a zero bid price and require each option series to have at least 385 valid quote minutes per day.

In our logit specifications, the likelihood of early exercise is related to the moneyness and maturity. Our operational measure of moneyness is related to the probability that the option will still be in-the-money at maturity and this is proportional to \sqrt{T} . We measure this by $(X/S)/\sqrt{T}$ for call options and by $(S/X)/\sqrt{T}$ for put options. Scaling the moneyness by \sqrt{T} makes the measure more comparable across maturities. Since there is a wide variation in the daily minimum EBV, we use an indicator variable that is one if the daily minimum is at least five cents, and zero otherwise. For the persistence of boundary violations, we compute the number of minutes during the trading day that an option exhibited a boundary violation. We also include the option's volume, open interest, and implied volatility as control variables. For the put models, we include an additional indicator variable that is one if the stock price is below the critical level where the put should be exercised early, and zero otherwise.

The results are presented in Table 5. The first three columns contain the estimates for call options, while the last three columns provide the put option estimates. As expected, the results in column 1 show that the likelihood of early exercise for call options is strongly related to our moneyness measure. The interesting result, however, is that both EBV variables are highly

significant. All else constant, the existence of EBVs and their duration, strongly predict early exercise. Of the control variables, open interest and daily trading volume are positively related to the exercise likelihood, while the implied volatility is insignificant.

The results are similar for the put options, as shown in column 4. Both EBV variables are again highly significant as is the early exercise price boundary variable, suggesting that exercise is more likely when the price is below the early exercise price barrier. Open interest is positively related to the exercise likelihood while the implied volatility and trading volume are insignificant.

One notable difference between the call and put results is that the moneyness coefficient for the puts is insignificant. One possible reason for this is that these puts have two desirable hedging properties: they have negative exposure to the underlying stock and virtually no gamma. If the puts are being used to hedge a long exposure to the underlying stock, then their exercise would require delivering the stock and liquidating the hedge, which may well not fit with the investor's long-term investment plan.

Table 5 shows some interesting differences between customer and market maker exercises. Comparing columns 2 and 3, the coefficients for the EBV variables are significantly larger for the market makers, suggesting that they respond more aggressively to EBVs than do the customers. Similarly, the coefficients in columns 5 and 6 show that market makers are more responsive than customers to both the EBVs and the early put exercise price barrier.

A potential concern is that our results may be influenced by liquidity differences related to price levels. As discussed earlier, the minimum bid-ask spread is \$0.10 for option prices above \$3.00 and \$0.05 for prices below \$3.00. As a robustness check, we tried adding two indicator variables

to the logit model from Table 5. The first variable was one if the option price was between \$2.50 and \$3.00, and zero otherwise, while the second was one if the price was between \$3.00 and \$3.50 and zero otherwise. Five of the six sub-samples failed to reject the hypothesis of equality. We also reestimated the Table 5 specifications for the pilot and non-pilot samples separately. The relation between EBVs and likelihood of early exercise for both sub-samples was similar to the results from Table 5. Finally, as a robustness check, we included the bid-ask spread of the option as a proxy for short sale costs. The results from Table 5 were unchanged, and the coefficients on the bid-ask variables were generally negative.

Overall, our results suggest that early exercise is a rational response to EBVs and we need to revisit the claim that American-style options on non-dividend paying stocks should not be exercised before maturity. Further, our findings illustrate the importance of incorporating EVBs into option pricing models. If option investors are aware of EBVs and try to avoid or minimize them, this will be reflected in option liquidity and the early exercise premium. We discuss these issues in the next section.

7. EBVs and the Measurement of the Early Exercise Premium, Implied Volatilities, and Market Liquidity

7.1 Boundary Violations and the Early Exercise Premium

Boundary violations in real world markets make exercise the rational way to unwind an in-the-money American option position for a much broader range of maturity and moneyness conditions than would be expected in a frictionless market. The economically meaningful increase in liquidity that American exercise provides relative to a comparable European option

should be reflected in an early exercise premium that is on top of any premium related to future dividend payout, and empirical research supports this view.

Figlewski (2017) develops an alternative American option pricing model that takes EBVs into account. Two assumptions are added to a standard Black-Scholes setting. First, the market bid price for a call is always the Black-Scholes value less half the bid-ask spread, which is modeled as a fixed minimum plus a constant times the amount the option is in the money. This induces EBVs for ITM calls.

Secondly, the position may need to be liquidated before expiration. There is a certain probability that the option will be held to maturity, and a constant conditional probability of liquidation on any prior date t , given that the investor has not liquidated before t . In this framework it is possible to compute the dollar value of American exercise in closed-form using a replication strategy.

As an illustration, we considered the additional value of American exercise for a call using the following parameter values, that are roughly comparable to those found in our data sample:

Stock price = 100; Exercise price = 80; Maturity = 30 days; Volatility = 40%; Interest: 5%;

Probability of liquidation before expiration: 75%, with constant intensity per day; no dividends.

Extensive empirical work was not done, but rough estimates of the bid-ask spread parameters were computed for a small number of stocks, ranging from very active ones like Exxon-Mobil and Facebook, to less active ones like T-Mobile and TripAdvisor. Although they differed greatly in their spread behavior, all were among the top 20% in trading volume in the population of traded options. Under these assumptions, the liquidity value of American exercise for an 80-

strike call was \$0.05, \$0.39, or \$0.89 for a high, medium, or lower (but still top 20%) option volume stock.

For comparison, the average dividend yield on a stock in the S&P 500 is about 2% annually, or \$0.50 a quarter on a \$100 stock. Consider a 30-day American call with the same terms as above on a stock that will go ex-dividend by \$0.50 in 15 days. An American call in this case would be worth about \$0.23 more than the equivalent European call. Thus the possibility of liquidity-based early exercise adds substantial value to an American call. Moreover, this is *in addition* to the value of exercise to get a dividend.

7.2 Boundary Violations and Mark-to-Market Option Valuation

It is customary to consider an option's market price to be the midpoint between the current best bid and ask prices. But one might argue that when the market bid price for an American option is an EBV, the lowest portion of the bid-ask spread becomes somewhat irrelevant to the value of the option, since the investor can realize a larger value by exercise. The most obvious case is when the bid is so low that the midpoint is also below intrinsic value. For the purpose of setting the required margin on an option position, it is questionable whether it is appropriate to value the option at the bid-ask midpoint when a higher value could be obtained by immediate exercise. In such a case, it could make sense to replace the EBV bid quote with the option's intrinsic value.

7.3 Boundary Violations and Implied Volatilities and Other Implicit Parameters

Boundary violations have implications for the calculation of implied volatilities and option hedge parameters, as well as for the implicit parameters which arise in more elaborate models that extend Black-Scholes.

An option's implied volatility is generally computed from the bid-ask midpoint, which assumes the midpoint is an unbiased estimate of the true option price. But since a call must be priced above intrinsic value in a theoretical model, implied volatility is undefined in the Black-Scholes model at an EBV bid price, as are the implied parameters in more sophisticated pricing models. When the quote midpoint is also below intrinsic value, IV and other implied parameters are unavailable. In this case, also, it might be better to replace the EBV bid price with the option's intrinsic value.

7.4 Boundary violations and liquidity measures.

Boundary violations have important implications for measures of transaction costs. One interpretation of a boundary violation is that the traditional bid-ask spread is too large. A more realistic 'adjusted' estimate of the bid-ask spread facing investors would be the difference between the ask price and the maximum of either the bid price, or the intrinsic value.

The difference between the traditional and boundary adjusted spreads can be substantial. For the deep in-the-money near-term call options in Table 1, the average traditional spread is about \$0.72 while the boundary adjusted spread is only \$0.38. The difference is smaller, but still substantial, for near-term Near ATM calls: \$0.33 for the traditional spread and \$0.26 for the boundary adjusted spread.

Many studies have used option bid and ask prices to make inferences about regulatory issues, market making costs, and option price dynamics. While it is uncertain how much these analyses are affected by boundary violations, given their prevalence, the violations are surely a material component of the data. An important implication is that it would be comparatively easy for a retail broker to offer option investors significant price improvement simply by providing a real

time comparison of the payoffs to exercising vs. selling their options. Alternatively, it should be straightforward for an algorithm to automatically route the customer's order to the best alternative, which would include early exercise. In our conversations with option broker-dealers some confirmed that they inform their customers when their trades would be sub-optimal.

8. Conclusion

Numerous studies have noted that options are often exercised early and concluded that investors behave irrationally. In this study, we pay careful attention to intraday option quotes and reach the opposite conclusion: early exercise is often the rational choice for options investors. The reason for this counter-intuitive result is simply that an option's bid price is frequently *less* than the option's intrinsic value. In these situations, selling at the bid price would yield fewer dollars than exercising the option and thus exercise becomes the rational choice.

We define an exercise boundary violation (EBV) as a situation where an option's bid price is low enough that an investor would be better off exercising his option instead of selling it. Previous research has noted the existence of boundary violations; our paper provides the first systematic analysis of EBVs. Using a sample of all U.S. stock options from March 2010, we show that boundary violations are common, persistent, and economically significant. Across all in-the-money calls, 48.6% of option quotes exhibit EBVs, with an average violation of \$0.31.

We find that option investors respond rationally to EBV quotes. In the cross-section of individual options, we find that the existence and duration of exercise boundary violations strongly predict the likelihood of early exercise. This holds for both calls and puts. Similarly, option investors are generally able to avoid selling their options at sub-optimal prices. In cases

where the bid price is below the option's intrinsic value, we find only a small fraction of trades occur at prices below intrinsic value.

The prevalence of boundary violations has important implications for option pricing. In theory, the only incremental value to an American call option above a European option is the ability to exercise just before an ex-dividend date. However, the American option's ability to avoid the EBVs gives it a source of value that is independent of dividends and it may help explain the puzzle that American options appear to be too expensive when compared to European options.

More generally, boundary violations are inconsistent with the assumptions made in traditional option pricing models, in which option quotes and trades should never be below intrinsic value. Yet, these events are commonplace. Liquidity issues have been shown to be important in other areas of finance. In the options market, these issues cannot be reduced to "noise" around theoretical relationships which may be ignored in empirical work. Liquidity constraints can lead to quite different behavior in practice than what may be optimal in theory. New models that can predict market behavior consistent with the data are needed.

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Table 1
Summary Statistics for Option Quotes

This table contains summary statistics for all option quotes during March 2010. Quotes from options with less than 180 observations per day or quotes with a zero bid price are excluded. Total EBV quotes is the number of quotes that exhibit exercise boundary violations. % Quotes with EBV is the percentage of quotes that exhibit boundary violations. % Options with EBV is the fraction of options that have one or more boundary violations during a trading day. Unconditional EBV is the mean boundary violation, averaged across all quotes. Conditional EBV is the mean boundary violation averaged across all quotes with boundary violations. EBV Adjusted Spread is the bid-ask spread computed from the ask price and the maximum of either the bid price or exercise boundary price. In the column headings for calls, Deep ITM corresponds to call options with a strike price/stock price ratio of between .5 and .7. Mid refers to a stock price price/strike price ratio between .7 and .9, and Near refers to a ratio between .9 and 1.0. In the column headings for puts Deep ITM corresponds to put options with a strike price/stock price ratio of between 1.3 and 1.5. Mid refers to a stock price price/strike price ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

Panel (a): Call Options

Maturity	< 1 Month	< 1 Month	< 1 Month	2-4 Months	2-4 Months	2-4 Months	> 4 Months	> 4 Months	> 4 Months
	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>
Total Quotes	24,233,775	39,210,622	20,603,553	41,276,612	62,774,801	34,708,238	56,578,410	69,881,531	36,786,243
Total EBV Quotes	23,925,783	36,374,930	7,538,395	38,676,453	34,011,298	1,804,803	33,821,214	11,251,916	332,710
% Quotes with EBV	98.73%	92.77%	36.59%	93.70%	54.18%	5.20%	59.78%	16.10%	0.90%
% Options with EBV	99.00%	98.21%	57.40%	98.13%	73.15%	12.67%	74.98%	26.80%	2.28%
Option Bid Price	\$15.90	\$7.67	\$2.27	\$16.20	\$7.85	\$3.14	\$19.71	\$10.53	\$5.77
Option \$ Spread	\$0.72	\$0.58	\$0.33	\$0.85	\$0.61	\$0.34	\$1.02	\$0.73	\$0.46
Option % Spread	9.26%	15.05%	31.71%	10.21%	13.90%	20.23%	9.29%	12.82%	15.57%
Unconditional EBV	\$0.34	\$0.25	\$0.07	\$0.37	\$0.16	\$0.01	\$0.27	\$0.06	\$0.00
Conditional EBV	\$0.35	\$0.26	\$0.14	\$0.38	\$0.24	\$0.18	\$0.39	\$0.28	\$0.35
EBV Adjusted \$ Spread	\$0.38	\$0.33	\$0.26	\$0.48	\$0.45	\$0.33	\$0.75	\$0.67	\$0.46
EBV Adjusted % Spread	4.85%	8.60%	26.11%	5.75%	10.26%	18.90%	6.61%	11.20%	15.02%

Panel (b): Put Options

Maturity Moneyess	< 1 Month <u>Deep</u>	< 1 Month <u>Mid</u>	< 1 Month <u>Near</u>	2-4 Months <u>Deep</u>	2-4 Months <u>Mid</u>	2- 4 Months <u>Near</u>	> 4 Months <u>Deep</u>	> 4 Months <u>Mid</u>	> 4 Months <u>Near</u>
Total Quotes	12,146,014	27,155,723	19,520,667	22,905,184	46,400,832	33,022,460	33,962,640	54,848,071	35,199,807
Total EBV Quotes	11,788,040	24,348,361	7,062,711	17,565,888	19,510,454	1,386,092	8,931,651	3,301,589	208,339
% Quotes with EBV	97.05%	89.66%	36.18%	76.69%	42.05%	4.20%	26.30%	6.02%	0.59%
% Options with EBV	99.27%	97.14%	55.96%	90.09%	62.73%	10.35%	43.51%	13.00%	1.40%
Option Bid Price	\$12.81	\$6.90	\$2.10	\$14.15	\$7.45	\$3.05	\$19.51	\$10.75	\$5.89
Option \$ Spread	\$0.65	\$0.58	\$0.34	\$0.76	\$0.59	\$0.34	\$0.84	\$0.63	\$0.43
Option % Spread	12.23%	17.48%	35.09%	11.48%	14.89%	20.27%	8.94%	11.37%	14.04%
Unconditional EBV	\$0.30	\$0.24	\$0.07	\$0.26	\$0.12	\$0.01	\$0.11	\$0.03	\$0.00
Conditional EBV	\$0.31	\$0.25	\$0.14	\$0.30	\$0.22	\$0.20	\$0.30	\$0.33	\$0.39
EBV Adjusted \$ Spread	\$0.35	\$0.34	\$0.27	\$0.50	\$0.47	\$0.33	\$0.73	\$0.60	\$0.42
EBV Adjusted % Spread	6.52%	10.46%	29.20%	7.35%	11.67%	19.09%	7.18%	10.27%	13.56%

Table 2**Duration of Boundary Violations**

This table examines the persistence of exercise boundary violations for all options. Option Days represents the number of individual options with positive option trading volume on day t and complete intraday quote data for day t. 10:00 a.m. EBV Frequency is the fraction of quotes that show exercise boundary violations when measured at 10:00 a.m. Avg Size of 10:00 a.m. EBV is the mean exercise boundary violation at 10:00 a.m., conditional on observing a boundary violation at 10:00 a.m. Avg Duration of EBV is the average number of minutes after 10:00 a.m. the boundary violation persists. In the column headings for calls, Deep ITM corresponds to call options with a strike price/stock price ratio of between .5 and .7. Mid refers to a stock price price/strike price ratio between .7 and .9, and Near refers to a ratio between .9 and 1.0. In the column headings for puts Deep ITM corresponds to put options with a strike price/stock price ratio of between 1.3 and 1.5. Mid refers to a stock price price/strike price ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

Panel (a): Calls

Maturity Moneyess	< 1 Month	< 1 Month	< 1 Month	2-4 Months	2-4 Months	2-4 Months	> 4 Months	> 4 Months	> 4 Months
	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>
Option Days	64,497	104,737	55,087	110,416	167,730	92,759	151,442	186,581	98,357
10:00 a.m. EBV Frequency	99.56%	93.43%	35.77%	93.96%	53.51%	4.92%	59.51%	15.84%	0.93%
Avg Size of 10:00 a.m. EBV	\$0.38	\$0.29	\$0.19	\$0.40	\$0.30	\$0.28	\$0.46	\$0.39	\$0.41
Avg Duration of EBV	348	297	189	313	235	157	266	228	178

Panel (b): Puts

Maturity Moneyess	< 1 Month	< 1 Month	< 1 Month	2-4 Months	2-4 Months	2-4 Months	> 4 Months	> 4 Months	> 4 Months
	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>
Option Days	32,804	72,812	52,309	61,878	124,258	88,316	91,364	146,771	94,144
10:00 a.m. EBV Frequency	97.31%	90.46%	36.46%	77.41%	42.65%	4.35%	26.64%	6.27%	0.55%
Avg Size of 10:00 a.m. EBV	\$0.33	\$0.29	\$0.20	\$0.34	\$0.29	\$0.30	\$0.41	\$0.45	\$0.46
Avg Duration of EBV	335	296	196	274	216	154	207	188	219

Table 3
Summary Statistics for Option Trades

This table provides summary statistics for all in-the-money option trades during March 2010. Total Trades is the aggregate number of trades that occurred for each sub-group. % Trades<EBV is the percentage of trades that occur at a price below the exercise boundary price. % Volume<EBV is the percentage of volume that occurs at a price below the exercise boundary price. Trade Size is the average number of contracts for trades that occur at a price below the exercise boundary price. \$ Deviation per share is the average dollar amount per share from selling at a price below the exercise boundary price. \$ Deviation per trade is the average dollar amount per trade from selling at a price below the exercise boundary price. \$ Deviation Total is the cumulative amount for each sub group from selling at a price below the exercise boundary price. In the column headings for calls, Deep ITM corresponds to call options with a strike price/stock price ratio of between .5 and .7. Mid refers to a stock price price/strike price ratio between .7 and .9, and Near refers to a ratio between .9 and 1.0. In the column headings for puts, Deep ITM corresponds to put options with a strike price/stock price ratio of between 1.3 and 1.5. Mid refers to a stock price price/strike price ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

Panel (a): Call Options

Maturity Moneyness	< 1 Month	< 1 Month	< 1 Month	2-4 Months	2-4 Months	2-4 Months	> 4 Months	> 4 Months	>4 Months
	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>	<u>Deep</u>	<u>Mid</u>	<u>Near</u>
Total trades	19,030	179,246	1,148,077	17,433	161,612	702,221	38,459	119,368	212,439
% Trades<EBV	12.10%	12.13%	2.21%	12.40%	3.79%	0.14%	5.37%	0.82%	0.00%
% Volume<EBV	22.26%	28.06%	14.27%	40.73%	17.95%	0.57%	18.24%	37.30%	0.16%
Trade size	31.21	130.29	194.70	122.51	138.20	72.58	206.87	1457.57	108.78
\$ Deviation per share	\$0.09	\$0.04	\$0.03	\$0.05	\$0.04	\$0.06	\$0.04	\$0.03	\$0.04
\$ Deviation per trade	\$282.31	\$515.25	\$560.70	\$638.69	\$536.64	\$454.09	\$835.67	\$4372.59	\$482.89
\$ Deviation total	\$649,884	\$11,204,146	\$14,211,482	\$1,380,211	\$3,284,747	\$446,829	\$1,725,656	\$4,258,904	\$30,422

**Panel (b):
Put Options**

Maturity Moneyness	< 1 Month Deep	< 1 Month Mid	< 1 Month Near	2-4 Months Deep	2-4 Months Mid	2-4 Months Near	> 4 Months Deep	> 4 Months Mid	>4 Months Near
Total trades	5,732	46,101	542,766	5,770	48,804	307,684	12,297	47,104	108,016
% EBV Trades	15.49%	8.69%	1.18%	6.41%	2.11%	0.05%	0.02%	0.11%	0.01%
% EBV Volume	21.87%	6.52%	1.05%	3.03%	2.63%	0.08%	0.24%	0.12%	0.00%
Trade size	26.14	13.51	16.24	11.90	30.38	33.55	11.58	25.40	6.78
\$ Deviation per share	\$0.05	\$0.07	\$0.06	\$0.13	\$0.11	\$0.44	\$0.38	\$0.40	\$1.01
\$ Deviation per trade	\$126.03	\$89.09	\$96.19	\$156.69	\$334.63	\$1467.70	\$444.54	\$1027.19	\$683.56
\$ Deviation total	\$111,917	\$356,733	\$617,069	\$57,976	\$344,330	\$239,235	\$25,339	\$53,414	\$6,152

Panel (c): All Options

	<u>Calls & Puts</u>	<u>Calls</u>	<u>Puts</u>
Total trades	3,722,159	2,597,885	1,124,274
% Trades<EBV	2.01%	2.38%	1.16%
% Volume<EBV	11.52%	14.81%	0.98%
Trade size	148.50	176.09	17.30
\$ Deviation per share	\$0.04	\$0.03	\$0.08
\$ Deviation per trade	\$521.81	\$602.20	\$139.54
\$ Deviation total	\$39,004,446	\$37,192,281	\$1,812,165

Table 4**Summary Statistics for Option Exercises**

This table provides summary statistics for option contracts exercised before maturity during March 2010. An exercise event is when one or more contracts on an individual option are exercised on day t. Total Contracts Traded is the total trading volume on days when exercise events occurred. In Panel (c), Total contracts/total volume for puts that should be exercised is the ratio of exercise volume to trading volume on exercise event days. Near Term options have one month or less to maturity, Medium Term have two to four months, and Long term have at least five months to expiration. The Deep in-the-money calls have a strike price/stock price between .5 and .7, Mid options have a ratio between .7 and .9, and Near has a ratio between .9 and 1.0. For put options, Deep corresponds to ratio of between 1.3 and 1.5, Mid has a ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

Panel (a): Call Options

Maturity Category	<1 Month	< 1 Month	<1 Month	2-4 Months	2-4 Months	2-4 Months	>4 Months	>4 Months	>4 Months	Total
Strike Price Category	Deep	Mid	Near	Deep	Mid	Near	Deep	Mid	Near	
Exercise Events Total	2,289	4,893	1,313	833	581	62	258	104	11	10,344
Exercise Events Customer	1,108	3,402	1,106	479	428	47	154	84	8	6,816
Exercise Events Market Maker	1,384	1,912	285	401	175	17	110	22	3	4,309
Contracts Exercised Total	155,361	524,900	159,001	45,004	35,476	23,856	4,449	2,589	47	950,683
Contracts Exercised Customer	42,495	193,552	93,226	16,035	21,325	21,683	1,948	1,140	12	391,416
Contracts Exercised Market Maker	112,866	331,348	65,775	28,969	14,151	2,173	2,501	1,449	35	559,267

Panel (b): All Put Options

Maturity Category	<1 Month	< 1 Month	<1 Month	2-4 Months	2-4 Months	2-4 Months	>4 Months	>4 Months	>4 Months	Total
Strike Price Category	Deep	Mid	Near	Deep	Mid	Near	Deep	Mid	Near	
Exercise Events Total	619	2,291	861	257	262	17	9	6	2	4,324
Exercise Events Customer	170	720	378	48	59	11	4	6	0	1,396
Exercise Events Market Maker	467	1,693	566	212	207	7	5	0	2	3,159
Contract Exercised Total	49,079	128,659	64,186	11,590	13,178	649	167	75	21	267,604
Contracts Exercised Customer	10,733	35,829	16,607	2,047	1,927	76	15	75	0	67,309
Contracts Exercised Market Maker	38,346	92,830	47,579	9,543	11,251	573	152	0	21	200,295

Panel (c): Put Options that should be exercised early

Maturity Category	<1 Month	< 1 Month	<1 Month	2-4 Months	2-4 Months	2-4 Months	>4 Months	>4 Months	>4 Months	Total
Strike Price Category	Deep	Mid	Near	Deep	Mid	Near	Deep	Mid	Near	
Exercise Events Total	405	1,217	351	82	49	0	1	0	0	2,105
Exercise Events Customer	100	319	118	11	5	0	0	0	0	553
Exercise Events Market Maker	317	974	279	72	45	0	1	0	0	1,688
Contract Exercised Total	39,267	64,956	30,374	2,075	1,593	0	1	0	0	138,266
Contracts Exercised Customer	6,670	15,012	6,116	102	243	0	0	0	0	28,143
Total Contracts Traded	26,025	105,271	69,030	2,387	2,578	935	14	51	0	206,291
Total Contracts/Total Volume	150.88%	61.70%	44.00%	86.93%	61.79%	0.00%	7.14%	0.00%		67.02%

Panel (d): Put Options that should not be exercised early

Maturity Category	<1 Month	< 1 Month	<1 Month	2-4 Months	2-4 Months	2-4 Months	>4 Months	>4 Months	>4 Months	Total
Strike Price Category	Deep	Mid	Near	Deep	Mid	Near	Deep	Mid	Near	
Exercise Events Total	214	1,074	510	175	213	17	8	6	2	2,219
Exercise Events Customer	70	401	260	37	54	11	4	6	0	843
Exercise Events Market Maker	150	719	287	140	162	7	4	0	2	1,471
Contract Exercised Total	9,812	63,703	33,812	9,515	11,585	649	166	75	21	129,338
Contracts Exercised Customer	4,063	20,817	10,491	1,945	1,684	76	15	75	0	39,166
Contracts Exercised Market Maker	5,749	42,886	23,321	7,570	9,901	573	151	0	21	90,172

Table 5
The Likelihood of Early Option Exercise

This table provides estimates of logit models for the probability that an option is exercised before maturity. The dependent variable is one if an option is exercised on day t and zero otherwise. Moneyness for call options is $(X/S)/\sqrt{T}$. $EBV > 0_minutes$ is the number of minutes during the trading day that an individual option exhibits a boundary violation. $EBV > .05$ is an indicator variable if the minimum boundary violation for an individual option during day t is at least .05, and zero otherwise. Daily trading volume is the number of contracts on an individual option that are traded on day t, in 1000s. Open interest is the open interest for an individual option at the close of day t in 1000s. Implied Volatility is as of the close of day t. The moneyness variable for put options is defined as $(S/X)/\sqrt{T}$. Early exercise boundary is an indicator variable that is one if the price of the stock is sufficiently low that early exercise is optimal. The t-statistics for all models are adjusted for conditional heteroskedasticity and reported in parentheses.

	Total	Customer	Market	Total	Customer	Market
	Call Exercises	Call Exercises	Maker	Put Exercises	Put Exercises	Maker
			Call Exercises			Put Exercises
Constant	-6.617 (-111.94)	-6.662 (-113.16)	-8.794 (-44.25)	-7.667 (-48.395)	-7.919 (-55.94)	-8.843 (-34.08)
Moneyness	0.127 (34.03)	0.129 (32.84)	0.067 (14.25)	-0.018 (-0.313)	0.099 (12.07)	-0.036 (-5.40)
Minutes EBV > 0	0.012 (71.92)	0.011 (65.21)	0.016 (29.43)	0.015 (34.535)	0.012 (30.37)	0.018 (25.34)
EBV >= .05	1.066 (41.11)	0.681 (23.47)	0.928 (27.51)	0.883 (18.130)	0.333 (5.13)	0.672 (13.53)
Daily trading volume / 1000	0.059 (2.12)	0.058 (2.15)	0.024 (2.48)	-0.007 (-0.466)	0.019 (1.31)	-0.076 (-0.47)
Open interest / 1000	0.006 (3.04)	0.006 (3.01)	0.002 (0.91)	0.012 (5.800)	0.007 (2.42)	0.013 (4.72)
Implied Volatility	0.000 (-0.06)	-0.000 (-0.06)	-0.003 (-0.65)	-0.035 (-1.362)	0.004 (0.11)	-0.096 (-3.21)
Early exercise boundary				1.195 (25.810)	0.072 (0.98)	1.196 (22.17)
Pseudo R²	0.224	0.139	0.095	0.247	0.066	0.197
Observations	197,610	197,610	197,610	104,329	104,321	104,321

Figure 1: Percentage of quote minutes that imply an EBV greater than \$x: Near term expiration options

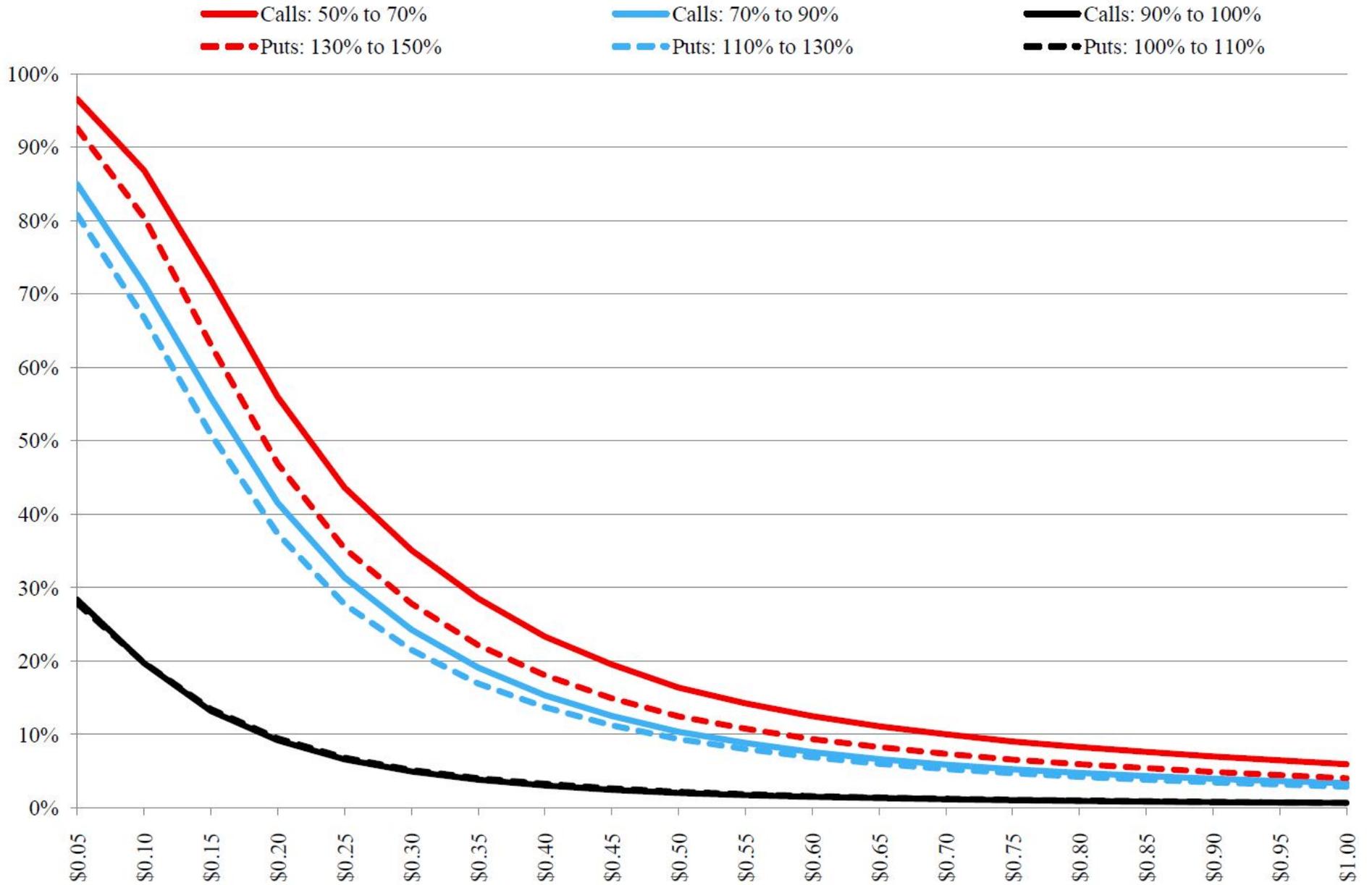


Figure 2: Percentage of quote minutes that imply an EBV greater than Sx: Middle term expiration options.

